Immiscible 2D Rayleigh-Taylor turbulence
Testing phenomenology with LBM simulations

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Introduction

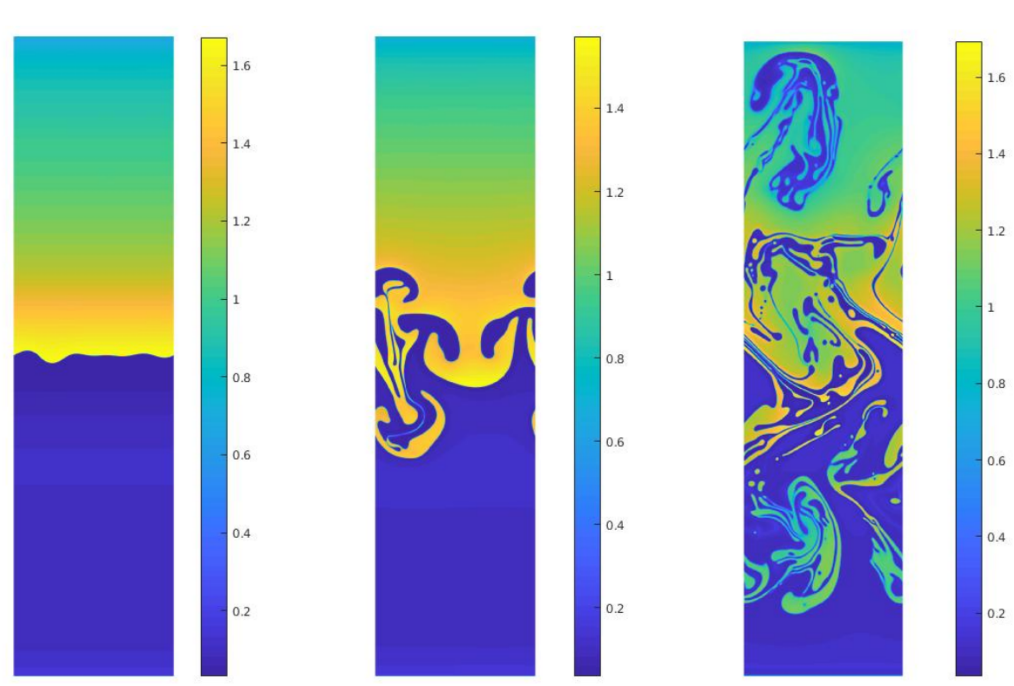
We simulate a turbulent motion induced by the **Rayleigh-Taylor instability for 2D immiscible two-phase flow**. For this purpose we use the multicomponent **lattice- Boltzmann method** with **Shan-Chen pseudopotential model implemented on GPUs**. The main advantage of this method is that it allows accurate simulations following highly sophisticated topological changes of the two-phase interface. We consider a **phenomenological theory** for 2D immiscible Rayleigh-Taylor turbulence, derived similarly to the 3D case studied by Chertkov, Kolokolov and Lebedev (2005). Then we test predictions of this theory with numerical simulations by measuring the growth of the **mixing layer**,**formation of drops** and analyzing the role of **interface** for energy transfer and mixing.

2D Immiscible Rayleigh-Taylor turbulence

We consider the Boussinesq system

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{\nabla p}{\rho_0} + \frac{\rho'}{\rho_0}\mathbf{g} + \nu\nabla^2\mathbf{u}$$
$$\nabla\cdot\mathbf{u} = 0.$$

to model the Rayleigh-Taylor instability. Surface tension leads to a formation of an **emulsionlike state** with **formation of drops**.



Phenomenology

From dimensional analysis we can estimate the **mixing layer** growth:

$$L \sim \alpha g t^2,$$

where α is a coefficient related to the fraction of potential energy converted into kinetic energy. Considering the Bolgiano-Obukhov theory and the time dependence of the mixing layer, we can estimate the **the velocity fluctuations**:

$$\delta u_r \sim (\alpha g)^{2/5} r^{3/5} t^{-1/5}.$$

The size l of the **typical drop** can be estimated from the scale at which the kinetic and surface energy densities are of the same order:

$$l \sim \left(\frac{\gamma^5}{g^4\rho^9}\right)^{1/11} t^{2/11}.$$

To find the **Kolmogorov scale** η we match $\delta u_\eta \eta \sim \nu$, therefore:

$$\eta \sim (\alpha g)^{-1/4}\nu^{5/8}t^{1/8}.$$

The number of drops N_l with typical size l can estimated by $N_l \sim \frac{LB}{l^2}$, where B is the horizontal length the fluid domain. Thus:

$$T_l \sim B\alpha g^{15/11}\rho^{9/11}\gamma^{-5/11}t^{20/11}.$$

Analogously, the drops with typical size $r \gg l$ have the total length T_r given by:

$$T_r \sim N_r \cdot r \sim B\alpha g \frac{t^2}{r}.$$

Schan-Chen pseudopotential method

In **Schan-Chen pseudopotential method** the intermolecular forces act between pairs of molecules and are additive. The simplest form of the **discretized Schan-Chen force for a multicomponent system** is represented through the sum:

$$F^{SC(\sigma)}(x) = -\psi^{(\sigma)}(x)\sum_{\overline{\sigma}} G_{\sigma\overline{\sigma}} \sum_i w_i \psi^{(\overline{\sigma})}(x + c_i\Delta t)c_i\Delta t,$$

where the indices $1 \leq \sigma, \overline{\sigma} \leq S$ are labels for the fluid components. To simulate multicomponent flows the Schan-Chen F_{sc} is inserted in the forcing term in the standard LBGK equation for each component.

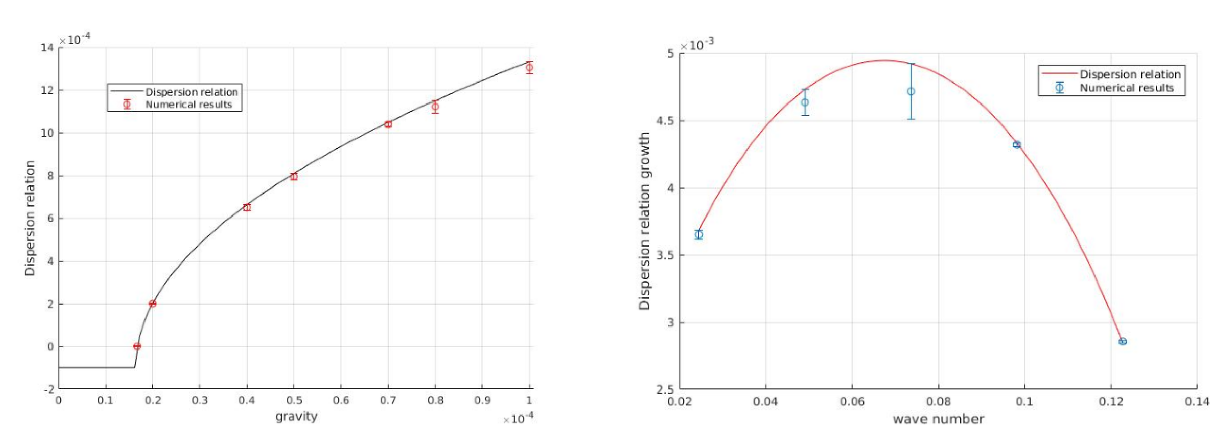
In our numerical experiments this method have been simulated parallelized on GPUs using the language CUDA C.

Numerical results

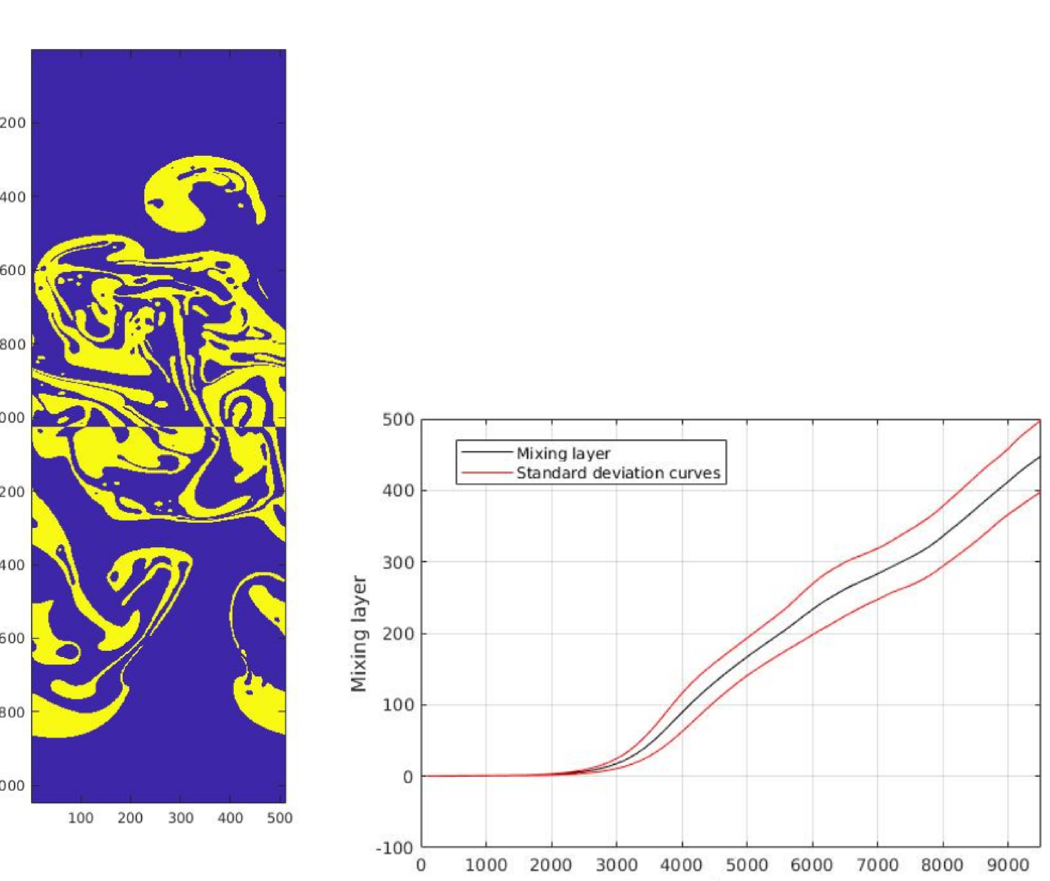
Suppose that we impose a small perturbation of the form $h(x,0) = h_0\cos(kx)$ on the planar interface $y = 0$ separating two fluids. The perturbation is expected to have a **exponential growth** $h \sim \exp(\omega(k)t)$, with

$$\omega(k) = -\nu k^2 + \sqrt{gk - \frac{\gamma}{2\rho_0}k^3 + (\nu k^2)^2},$$

where ν is the effective cinematic viscosity and γ is the surface tension.

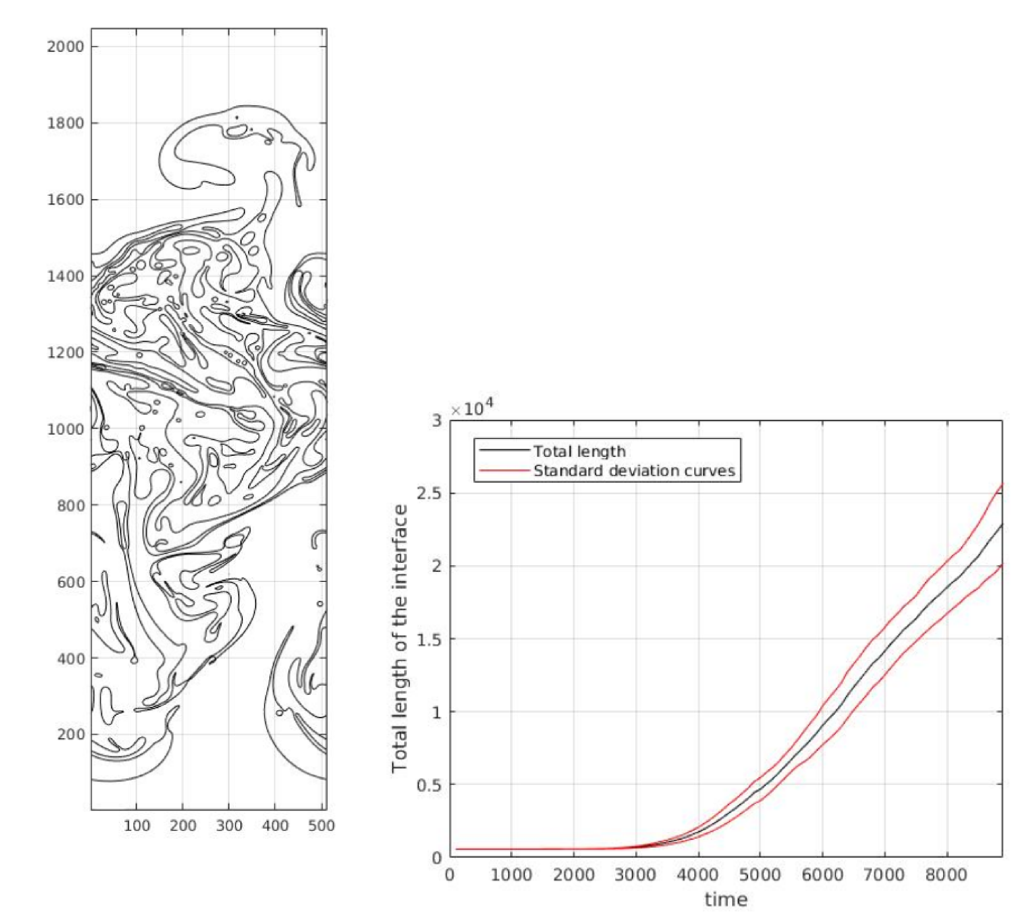


The mixing layer was calculated measuring how much the components penetrate each other.



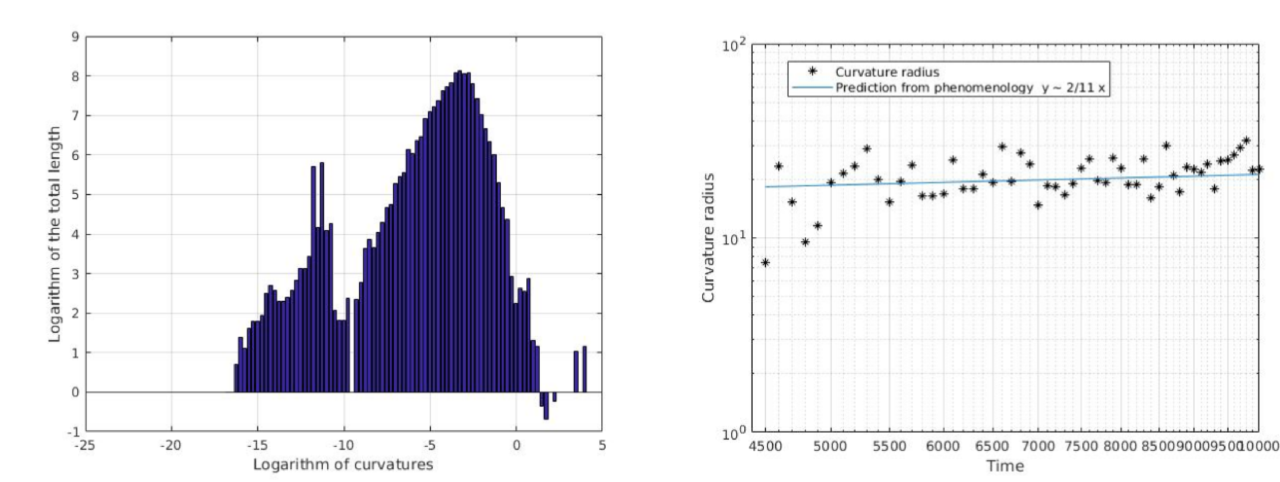
(a) Logical matrix for mixing layer. (b) Mean mixing layer for 25 simulations.

One of the main differences between the miscible and immiscible RT turbulence is the appearance of clear interfaces. We analyze the statistics of energy transfer and mixing of the interface.



(a) Interface points (b) Mean total length of the interface for 10 simulations.

For each point of the contour we can associate a length taking the average lengths of the segments adjacent to this point. We can also associate to the same points curvatures. For each interval of curvature values we have associated a total length. The maximum total length will give us information about the **typical size of the small structures**.



Conclusions

- Development of a **lattice-Boltzmann code** for simulations of **immiscible two-phase flows**.
- Development of a **phenomenological theory for immiscible 2D RT turbulence**, and verification of some predictions. In particular, the growth of the **mixing layer** and the **typical drop size**.

References

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