Nonlinear propagation of one-dimensional waves: some recent experimental results

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Nonlinear propagation of one-dimensional waves: some recent experimental results

Pierre Suret, François Copie, Alexey Tikan, Adrien Kraych, Alexandre Lebel, Rebecca El Koussaifi, François Gustave

Gennady El, Giacomo Roberti, Thibault Congy, Andrey Gelash, Dmitry Agafontsev, Vladimir Zakharov, Alexander Tovbis

Eric Falcon, Félicien Bonnefoy, Guillaume Ducrozet, Guillaume Michel, Annette Cazaubiel, Gaurav Pradehusai, Miguel Onorato

Université de Lille 1 Sciences et Technologies
CEMPI Centre Européen pour les Mathématiques, la Physique et leurs Interactions
PhLAM Physique des Lasers Atomes et Molécules
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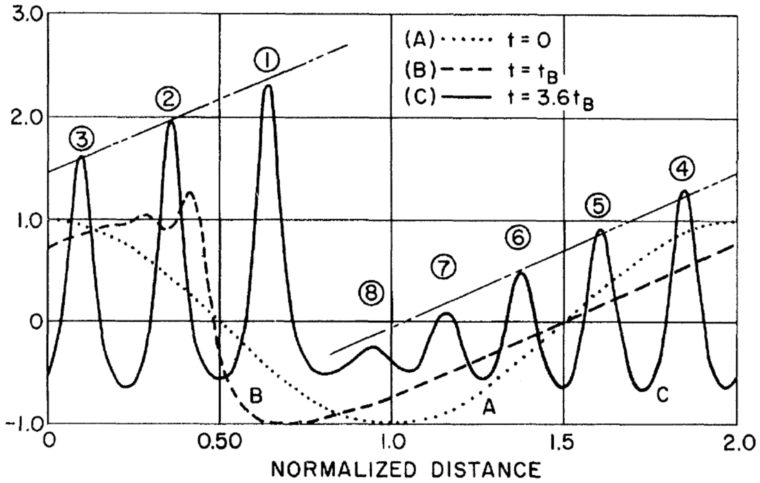
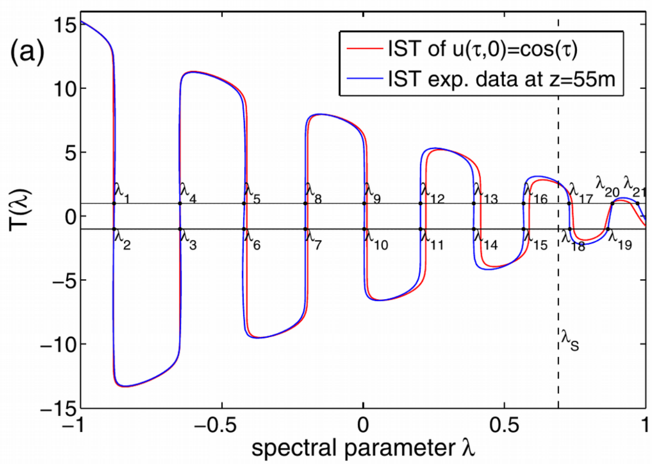
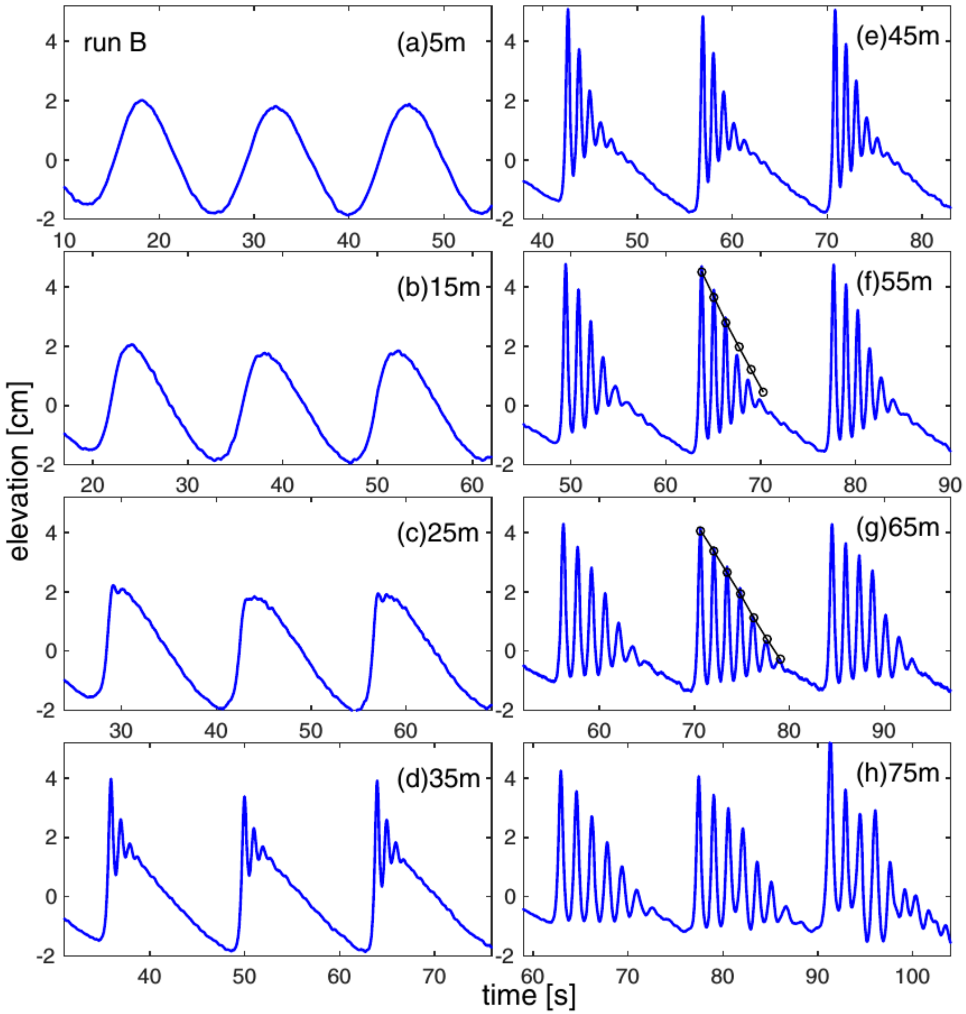
Introduction



N. Zabusky and M. Kruskal, PRL (1965)

$$u_\zeta - 6uu_\tau - \varepsilon^2 u_{\tau\tau\tau} = 0; \qquad u_0(\tau) = \cos(\tau)$$





S. Trillo *et al*, “Experimental Observation and Theoretical Description of Multisoliton Fission in Shallow Water”; Phys. Rev. Lett. **117**, 144102 (2016)

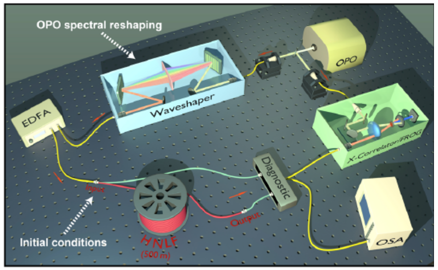
See also I. Redor et al, “Experimental evidence of a hydrodynamic soliton gas”, PRL **122,** 214502 (2019) and the talk by T. Congy for recent developments on the subject of soliton gas

Introduction

In optical fiber experiments: **Defocusing** propagation regime (Normal dispersion)

$$i\epsilon \frac{\partial \psi}{\partial \xi} + \frac{\epsilon^2}{2}\frac{\partial^2 \psi}{\partial \tau^2} - |\psi|^2\psi = 0$$

$$\psi(\tau,\xi) = \sqrt{\rho(\tau,\xi)}e^{i[\phi(\tau,\xi)/\epsilon]}$$ *Madelung Transformation*



$$\rho_\xi + (\rho u)_\tau = 0$$
$$u_\xi + uu_\tau + \rho_\tau = 0$$

ρ: fluid height/optical power

u: depth-averaged horizontal velocity/instantaneous frequency $u(\tau,\xi) = \phi_\tau$

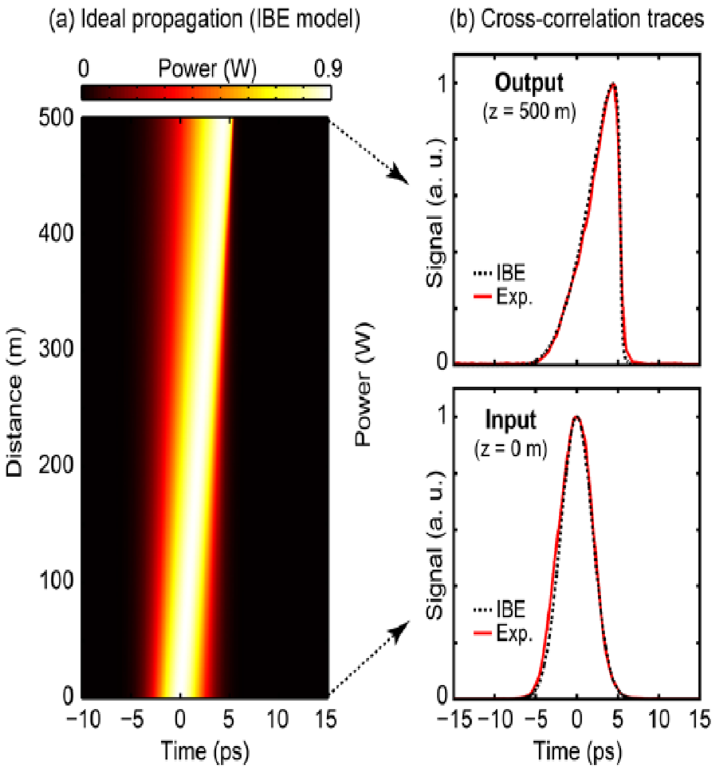
$$r_{1,2}(\tau,\xi) = u \pm 2\sqrt{\rho}$$ *Riemann invariants*

$$\frac{\partial r_{1,2}}{\partial \xi} + V_{1,2}\frac{\partial r_{1,2}}{\partial \tau} = 0$$

Propagation equations of two interacting Riemann waves

$$u_\zeta - 6uu_\tau = 0$$

Hopf Equation or Inviscid Burgers Equation



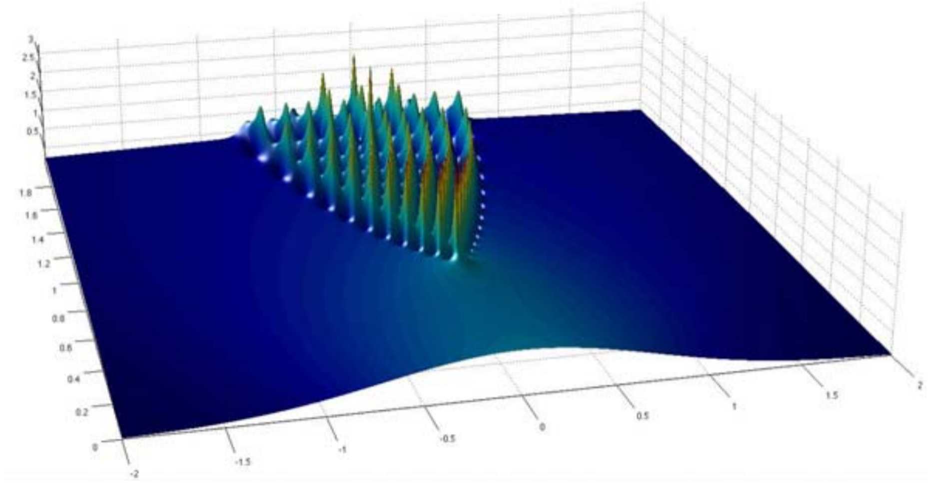
Random Riemann waves/Integrable turbulence
See the Poster by G. Roberti/T. Congy

B. Wetzel *et al*, PRL **117**, 073902 (2016)

Focusing propagation regime

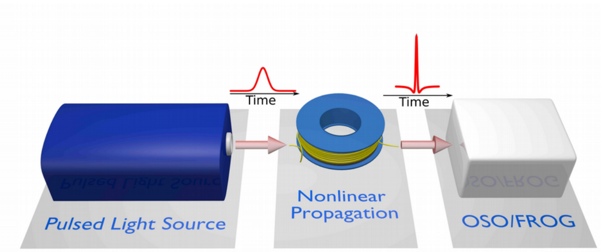
$$i\epsilon\frac{\partial\psi}{\partial\xi}+\frac{\epsilon^2}{2}\frac{\partial^2\psi}{\partial\tau^2}+|\psi|^2\psi=0$$

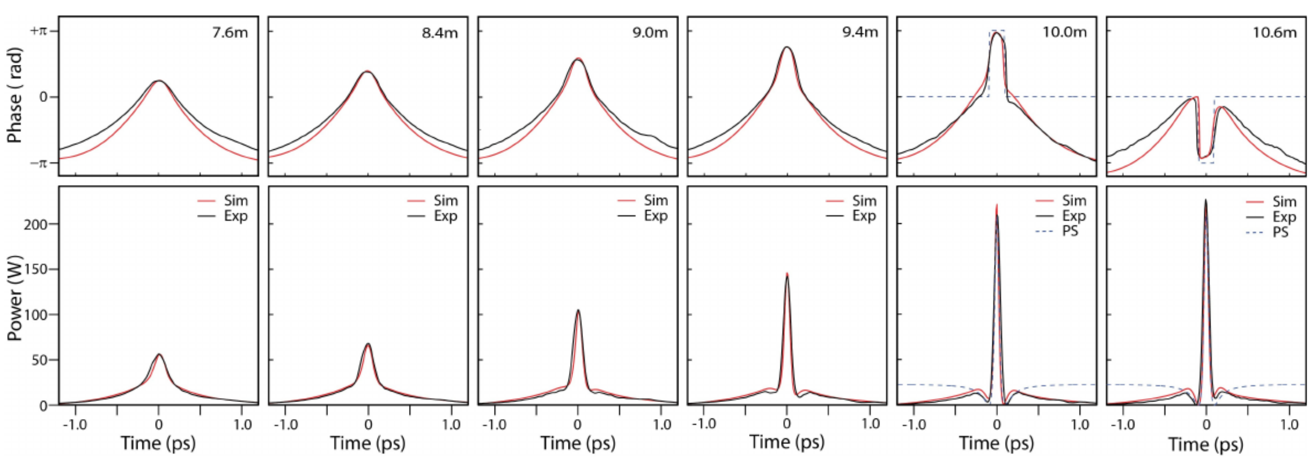
Self-focusing → gradient catastrophe → Peregrine soliton



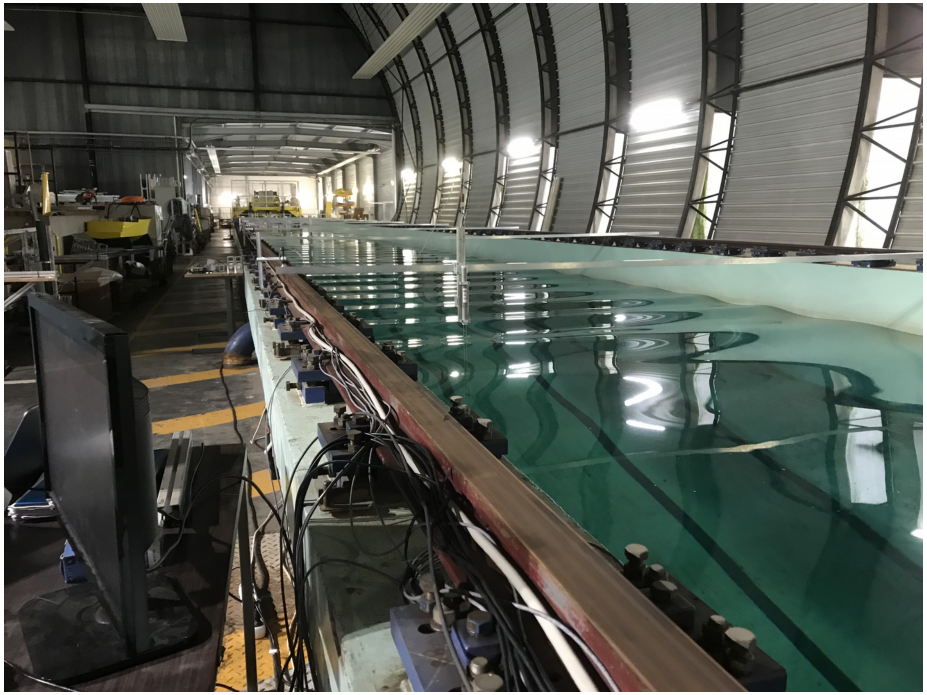
M Bertola, A Tovbis - Communications on Pure and Applied Mathematics Vol. LXVI, 0678–0752 (2013)

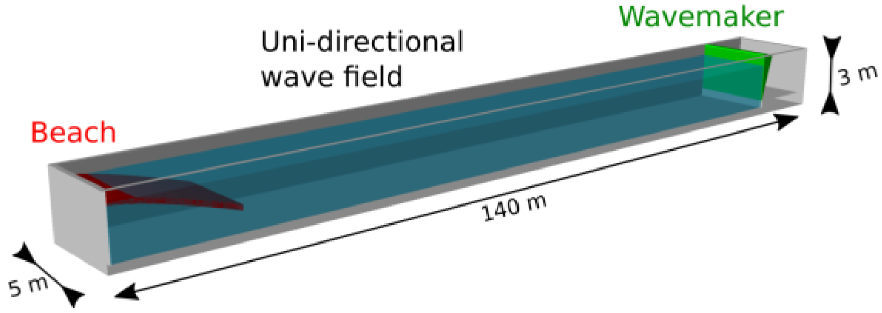
Optical fiber experiments:





A. Tikan *et al*, Phys. Rev. Lett. **119**, 033901 (2017)

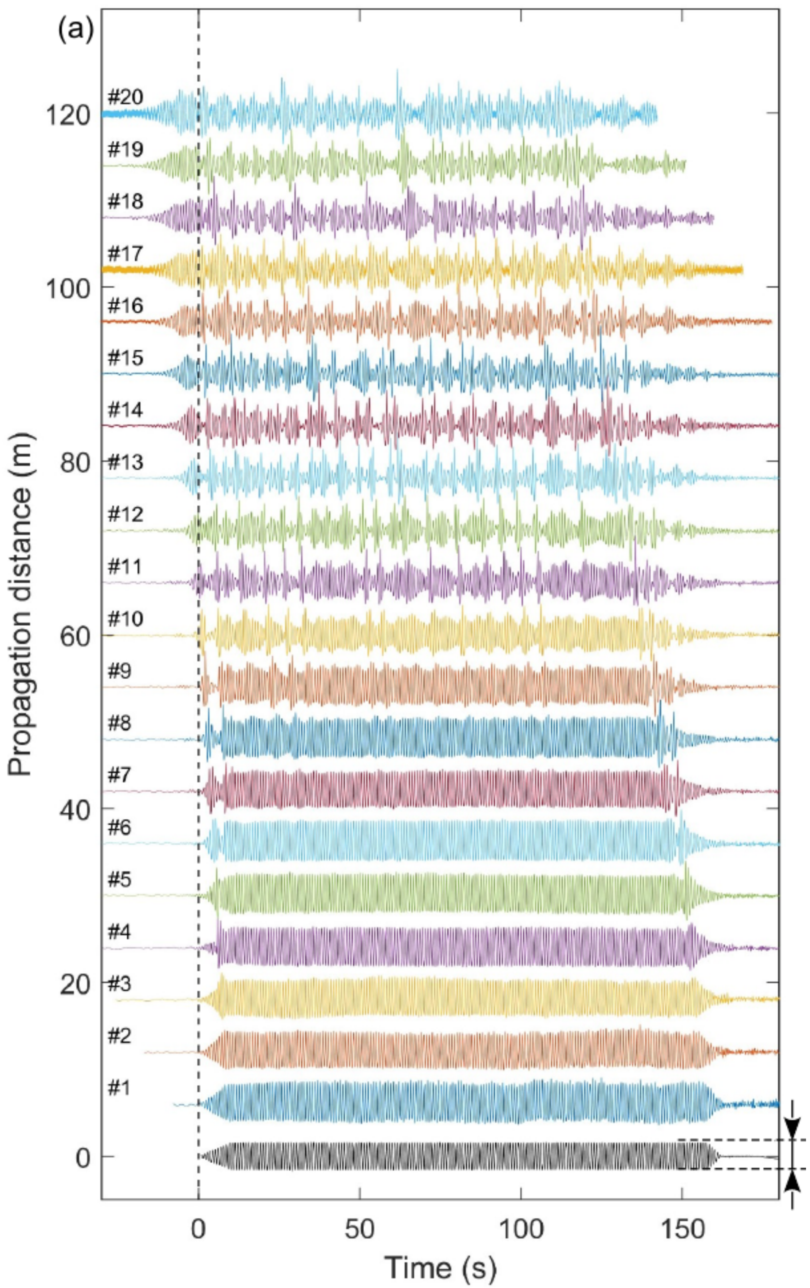




Experiments made in Ecole Centrale de Nantes (France)

Felicien Bonnefoy, Pierre Suret, Alexey Tikan, Francois Copie, Gaurav Prabhudesai, Guillaume Michel, Guillaume Ducrozet, Annette Cazaubiel, Eric Falcon, Gennady El, and Stephane Randoux

Benjamin-Feir instability



Benjamin-Feir instability

$k_0 h$ =12.3

Deep-water regime (focusing 1D-NLSE)

$$i\frac{\partial A}{\partial z} + \frac{k_0}{\omega_0^2}\frac{\partial^2 A}{\partial t^2} + k_0^3|A|^2 A = 0$$

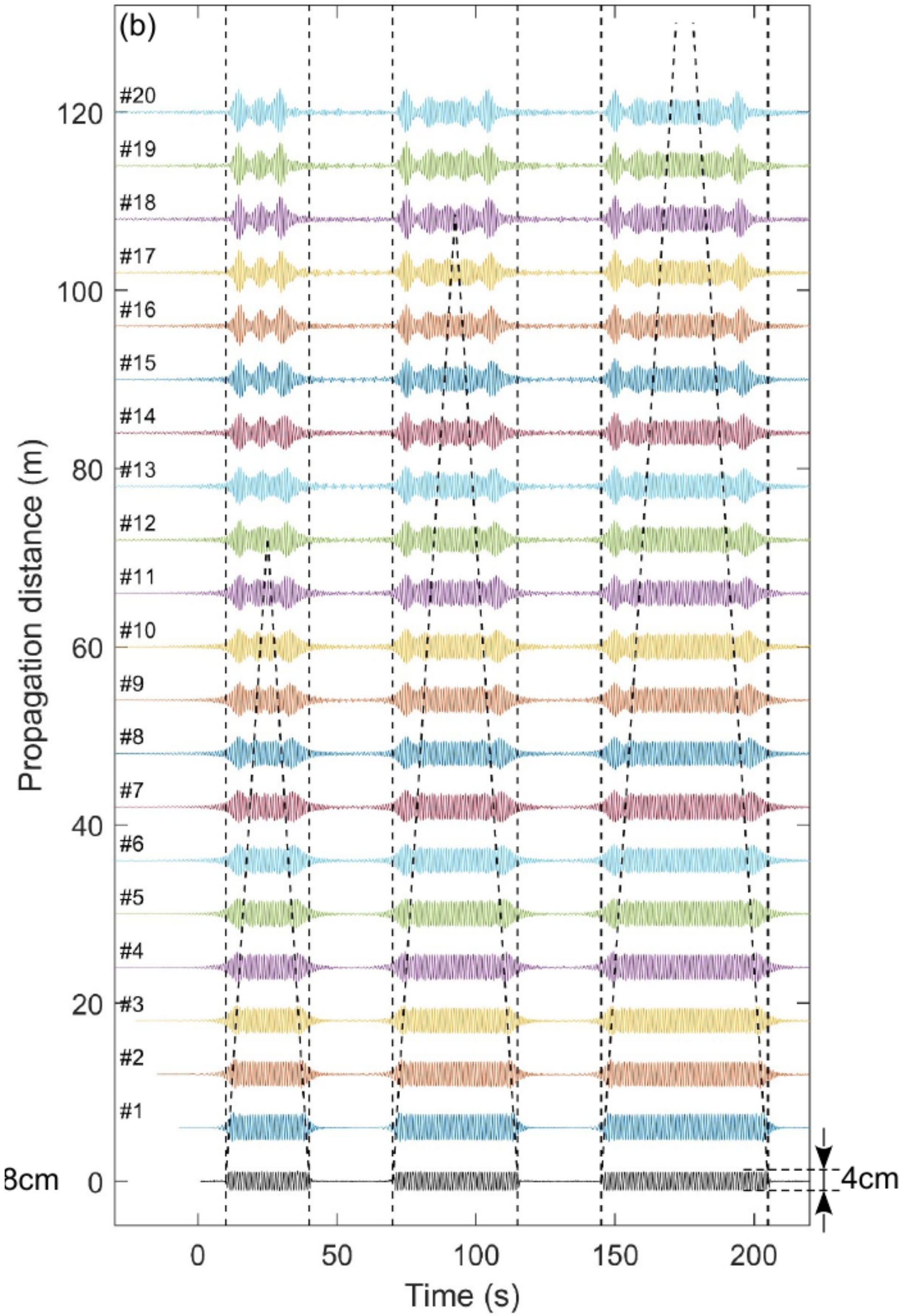
$$\omega_0^2 = k_0 g$$

steepness

$$k_0 a_0 \simeq 0.19$$

-T. B. Benjamin and J. E. Feir, Journal of Fluid Mechanics **27**, 417 (1967).
-T. B. Benjamin, Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences **299**, 59 (1967).
-Yuen, H.C.; Lake, B.M., "Instabilities of waves on deep water". Annual Review of Fluid Mechanics. **12,** 303–334 (1980)

Focusing dam break



Counterpropagating focusing dam break flows

Deep-water regime (focusing 1D-NLSE)

$$i\frac{\partial A}{\partial z} + \frac{k_0}{\omega_0^2}\frac{\partial^2 A}{\partial t^2} + k_0^3|A|^2 A = 0$$

$$\omega_0^2 = k_0 g$$

steepness

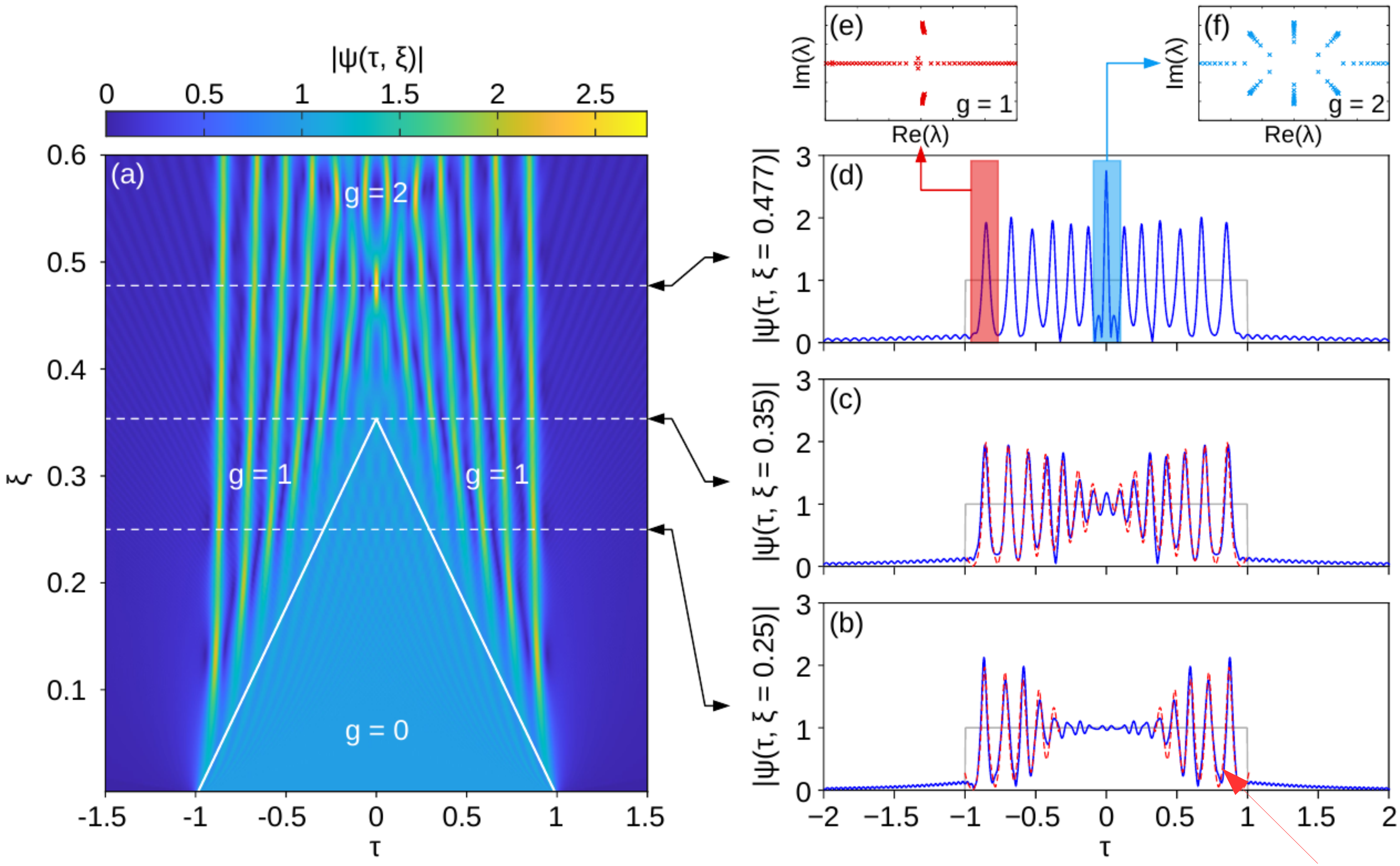
$$k_0 a = 0.082$$

The evolutions observed in the experiment can be interpreted within the framework of semi-classical theory of the 1D-NLSE

$$\epsilon = \sqrt{L_{NL}/L_D} \ll 1$$

$$i\epsilon\frac{\partial \psi}{\partial \xi} + \frac{\epsilon^2}{2}\frac{\partial^2 \psi}{\partial \tau^2} + |\psi|^2\psi = 0$$

Numerical simulations

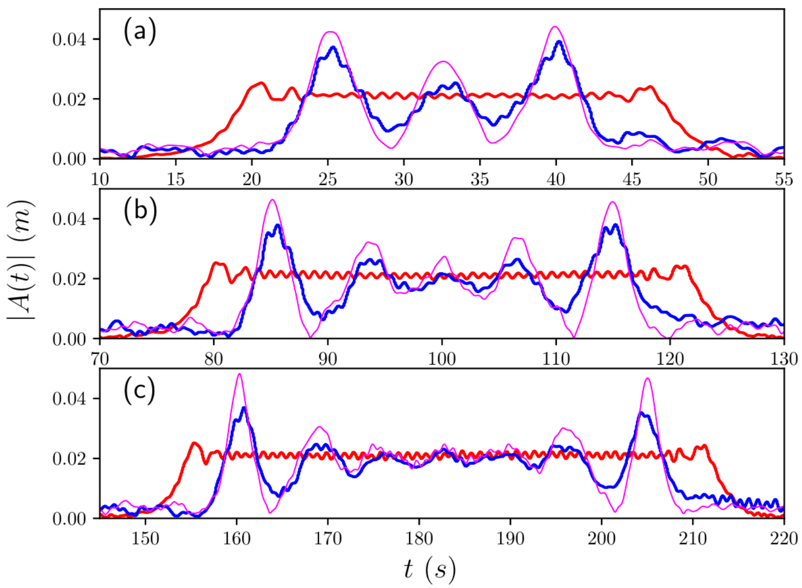


NLSE
BOX problem

$$\rho = (q+b)^2 - 4qb\,\mathrm{sn}^2\left(2\sqrt{qb/m}\,(\tau - a\xi - \tau_0)\varepsilon^{-1}; m\right)$$

$$i\epsilon \frac{\partial \psi}{\partial \xi} + \frac{\epsilon^2}{2}\frac{\partial^2 \psi}{\partial \tau^2} + |\psi|^2\psi = 0 \qquad \epsilon = \sqrt{L_{NL}/L_D} \ll 1$$

Dam break problem for the focusing nonlinear Schrödinger equation and the generation of rogue waves, G. El *et al*, Nonlinearity **29**, 2798 (2016)



Numerical simulations of the 1D-NLSE

$$i\frac{\partial A}{\partial z} + \frac{k_0}{\omega_0^2}\frac{\partial^2 A}{\partial t^2} + k_0^3|A|^2 A = 0$$

$$\epsilon = \sqrt{L_{NL}/L_D} \ll 1$$

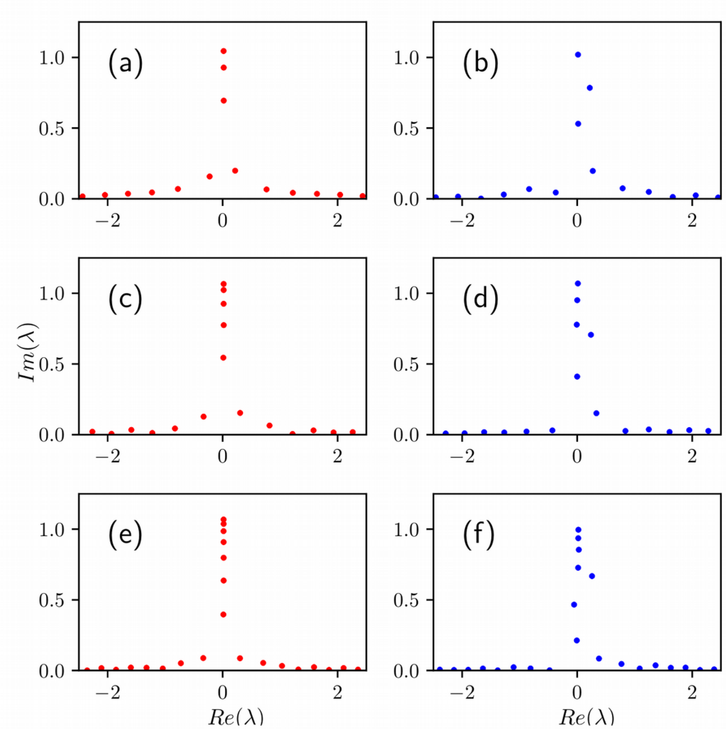
$$i\epsilon\frac{\partial \psi}{\partial \xi} + \frac{\epsilon^2}{2}\frac{\partial^2 \psi}{\partial \tau^2} + |\psi|^2\psi = 0$$

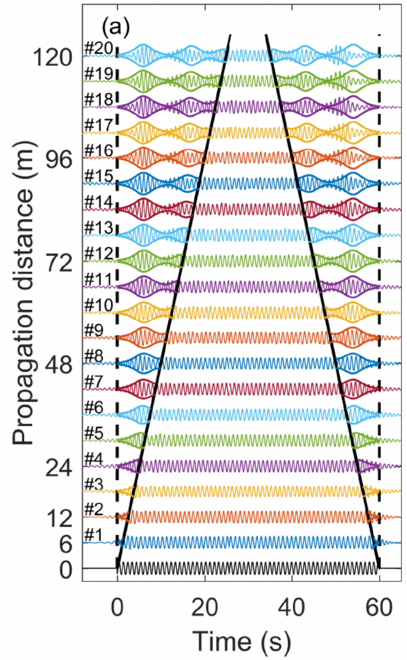
ΔT_j (s)	L_{NL} (m)	L_D (m)	ϵ	z_j^* (m)	N
30	39.86	4414	0.095	74	3
45	39.86	9932	0.063	111	5
60	39.86	17658	0.047	148	7

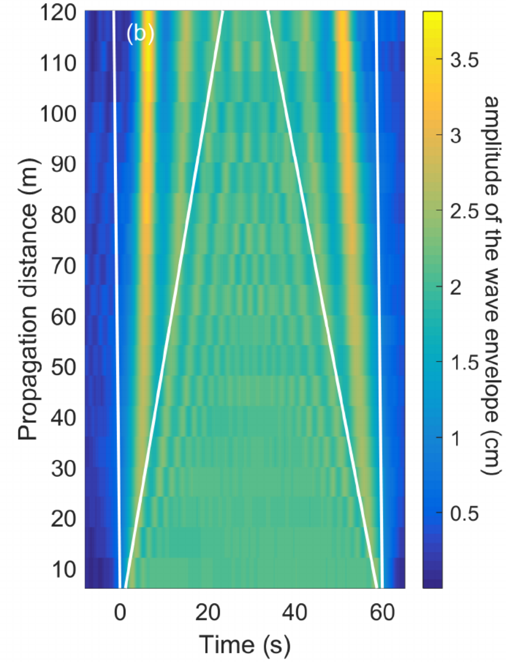
$$N = \mathrm{int}(1/2 + 1/(\pi\epsilon))$$

Nonlinear spectral analysis

$$\epsilon\frac{d\mathbf{Y}}{d\tau} = \begin{pmatrix} -i\lambda & \psi_0 \\ -\psi_0^* & i\lambda \end{pmatrix}\mathbf{Y}$$





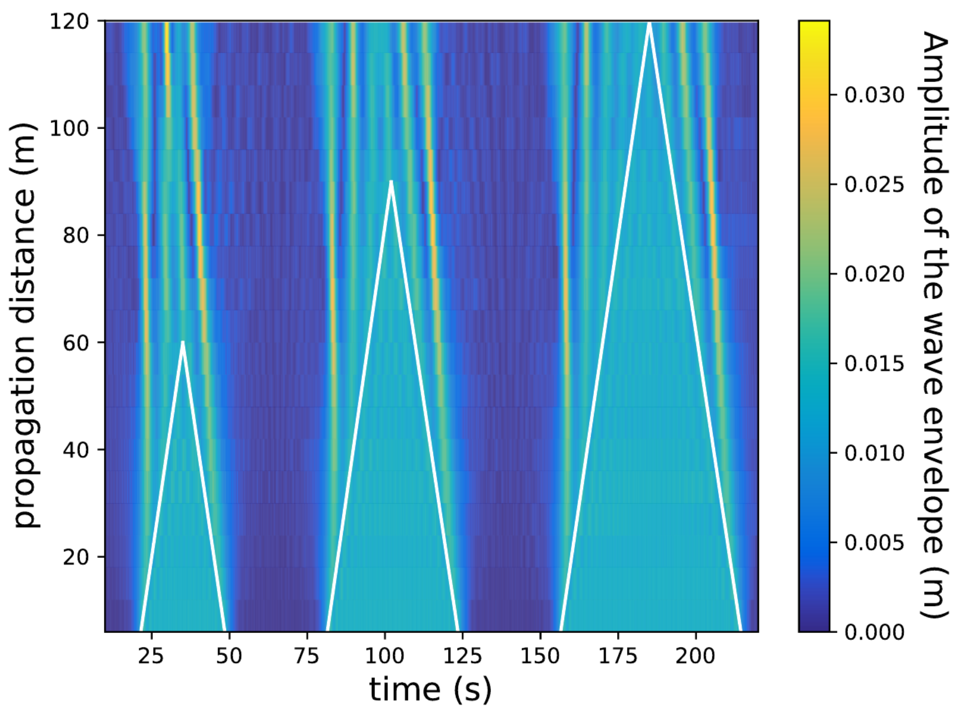


Breaking lines: $\Gamma_1: \qquad \xi = \dfrac{T - |\tau|}{2\sqrt{2}q}$

Modulated cnoidal envelopes

$$\rho = (q+b)^2 - 4qb\,\mathrm{sn}^2\left(2\sqrt{qb/m}\,(\tau - a\xi - \tau_0)\varepsilon^{-1}; m\right)$$

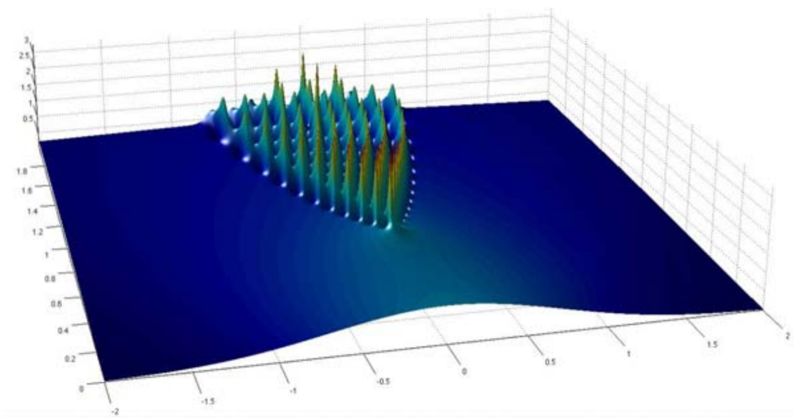
$$k_0 a = 0.082$$



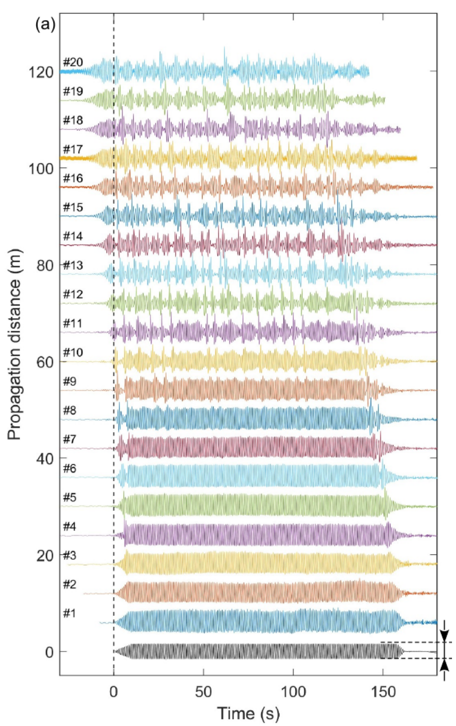
Robustness to higher-order effects

$$k_0 a = 0.09$$

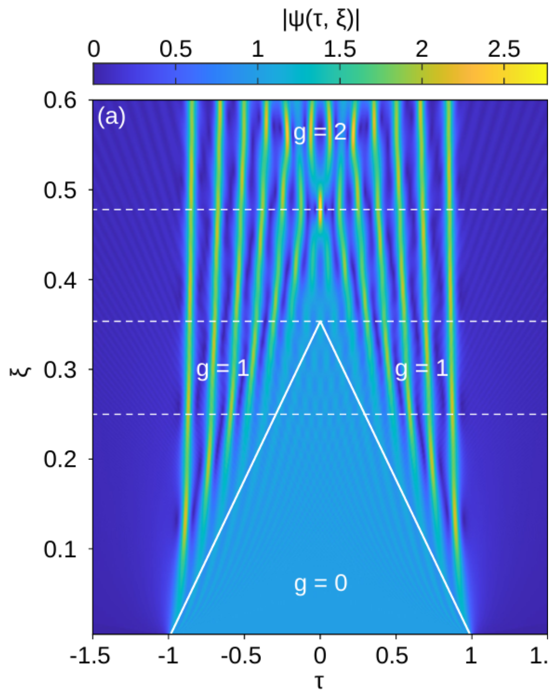
$$i\epsilon\frac{\partial\psi}{\partial\xi}+\frac{\epsilon^2}{2}\frac{\partial^2\psi}{\partial\tau^2}+|\psi|^2\psi=0$$



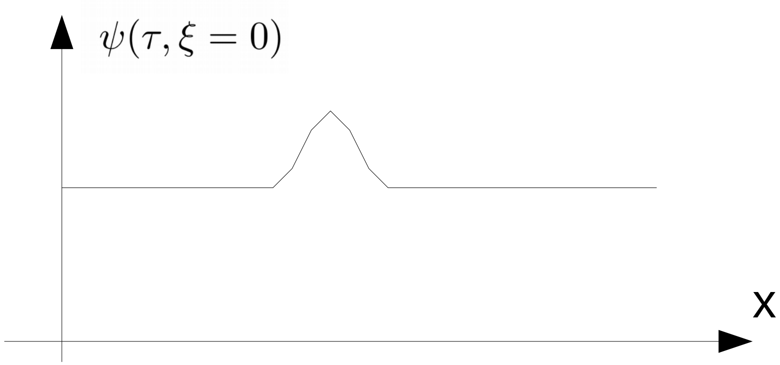
Self-focusing of broad wavepackets



Plane wave + noise: Benjamin-Feir instability



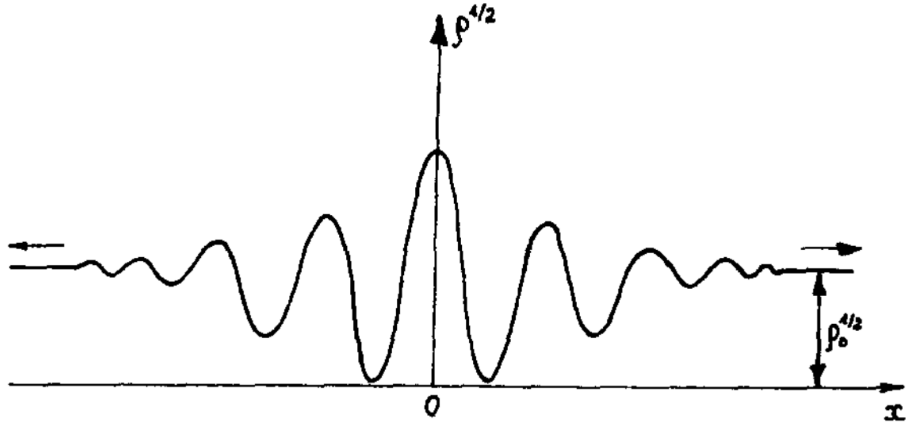
“Box-problem”: focusing dam break flows



Plane wave + localized perturbation

Localized perturbations

- Nonlinear stage of modulational instability: longtime asymptotic behavior of a localized perturbation of a constant background

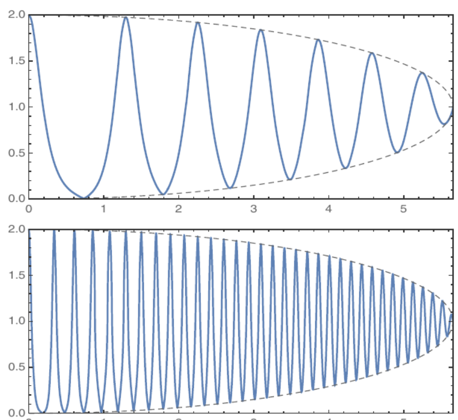


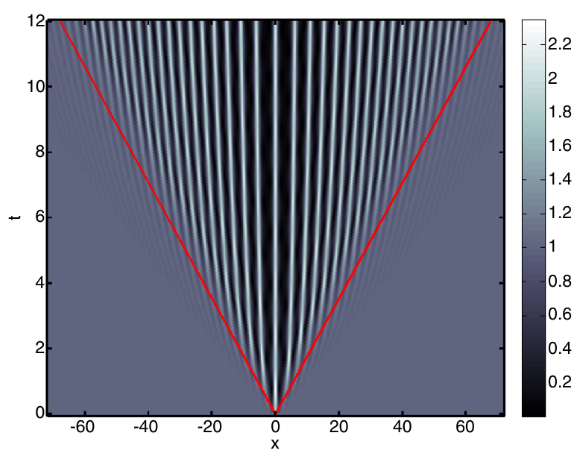
V. I. Karpman, J. Exp. Theor. Phys. Lett. **6**, 277 (1967).
V. I. Karpman and E. M. Krushkal, Sov. Phys. J. Exp. Theor. Phys. 28, 277 (1969).

G. A. El, A. V. Gurevich, V. V. Khodorovskiĭ, and A. L. Krylov, "Modulation instability and formation of a nonlinear oscillatory structure in a focusing medium", Phys. Lett. A **177**, 357 (1993).

Whitham modulation theory

$$|Q(x,t)|^2 = (q_0+b)^2 - 4q_0 b\,\mathrm{sn}^2\left(2\sqrt{q_0 b/m}\,(x - a\tau - x_0)\varepsilon^{-1}; m\right)$$



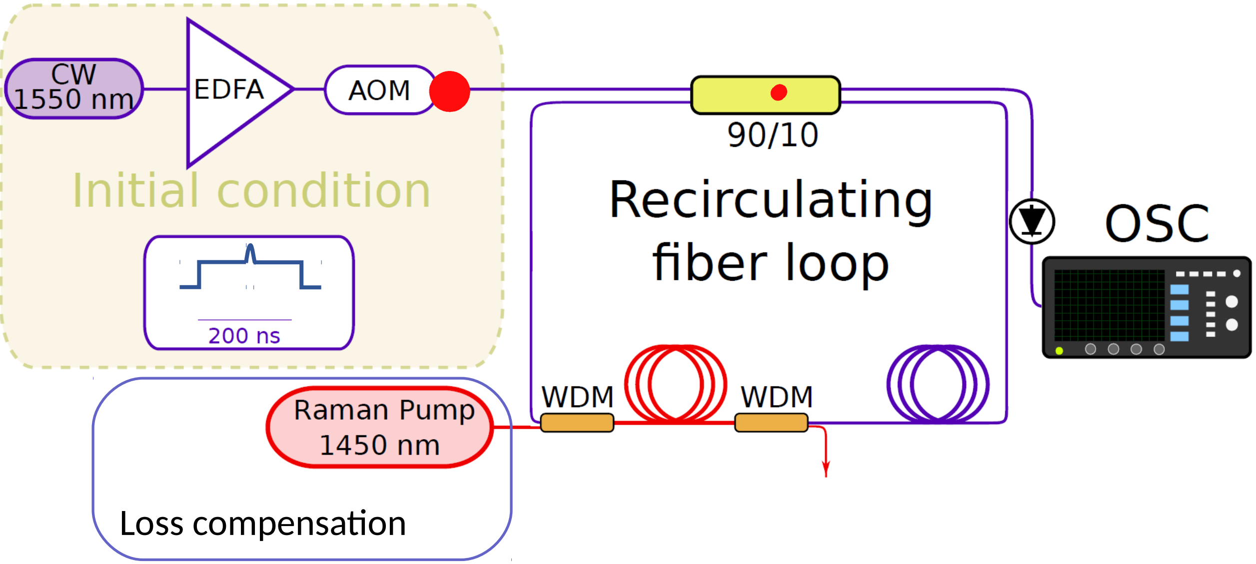


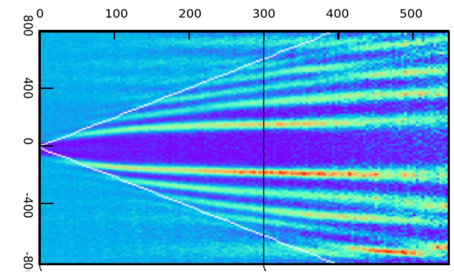
G. Biondini and D. Mantzavinos, "Universal nonlinear stage of modulational instability", Phys. Rev. Lett. **116**, 043902 (2016)

IST

Universal behavior in modulationally unstable media
G. Biondini, S. Li, D. Mantzavinos, and S. Trillo, SIAM Rev. **60**, 888 (2018)

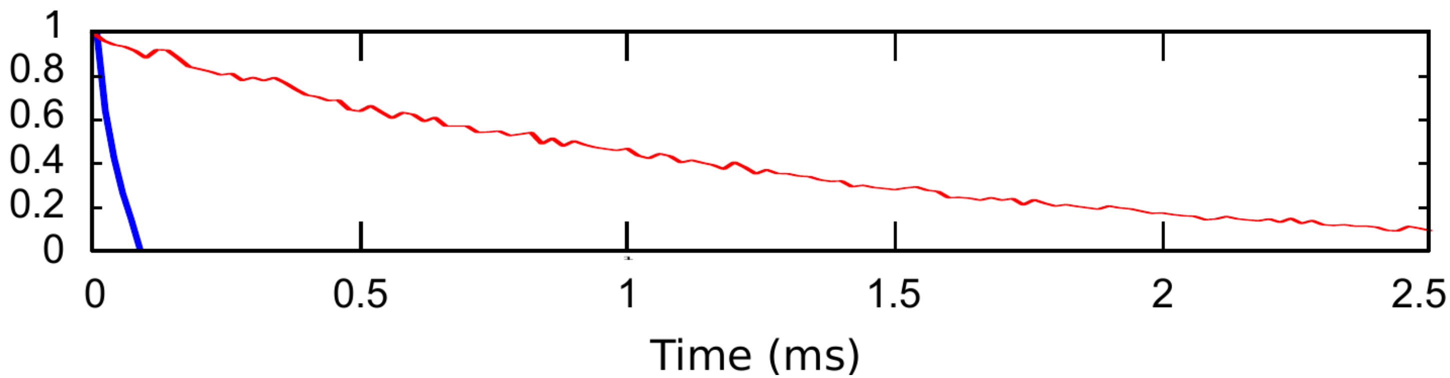
Localized perturbations





Reconstruction of the space-time diagram

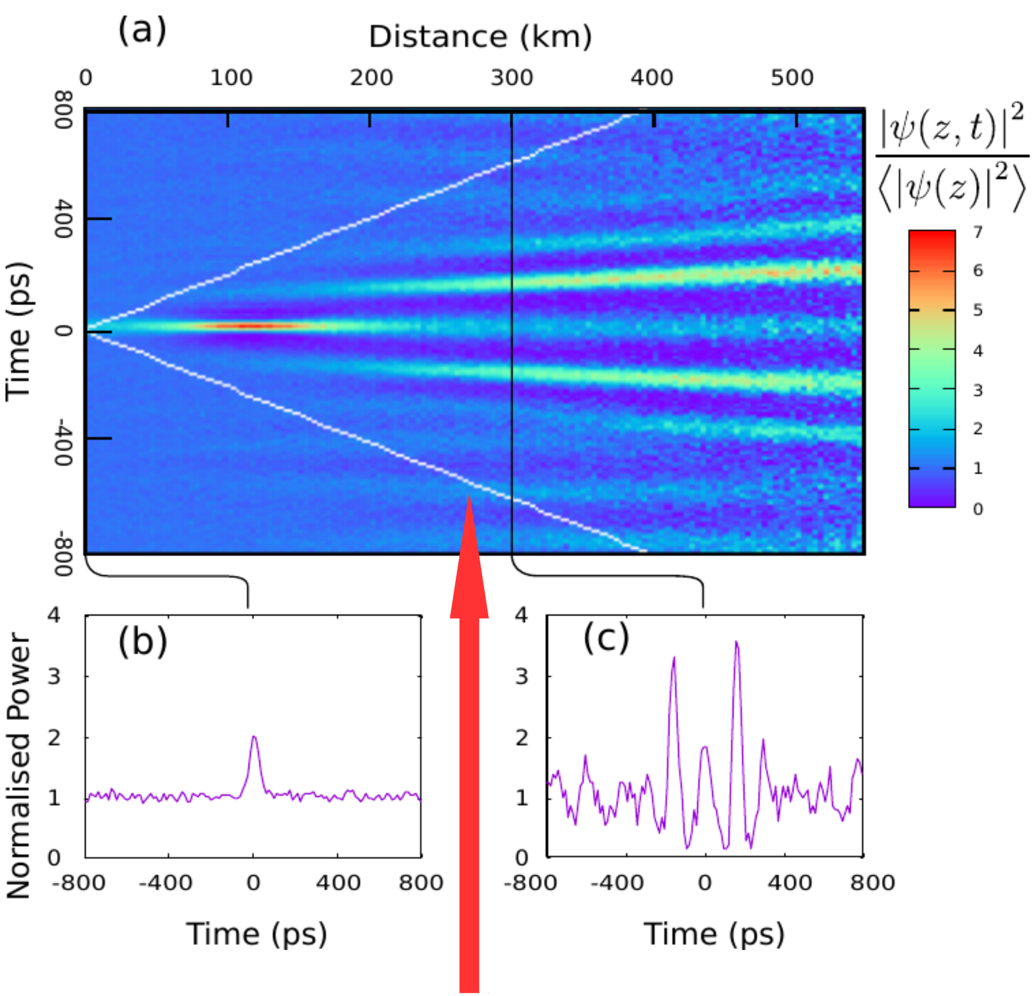
Cavity round trip time: 20 μs



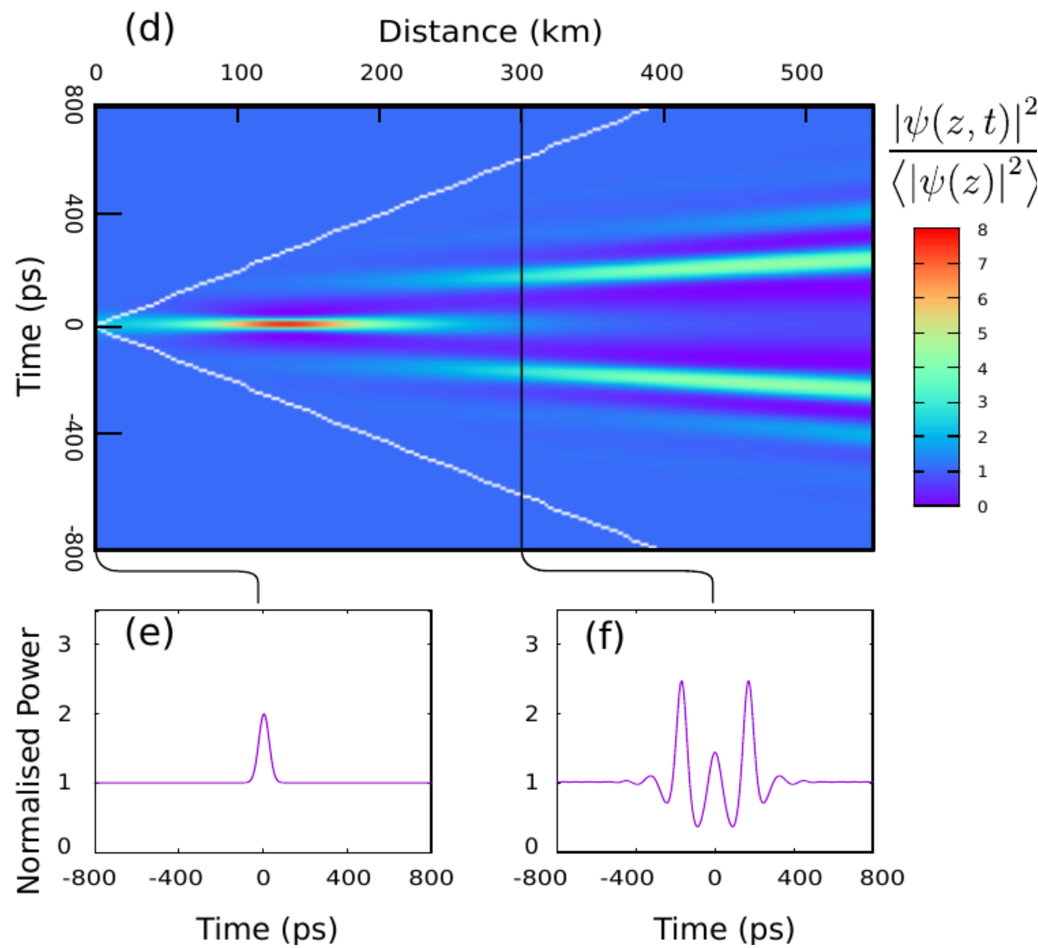
A. E. Kraych, P. Suret, G. El and S. Randoux, “Nonlinear Evolution of the Locally Induced Modulational Instability in Fiber Optics” Phys. Rev. Lett. **122**, 054101 (2019)

Localized perturbations

Experiments



Numerical simulations

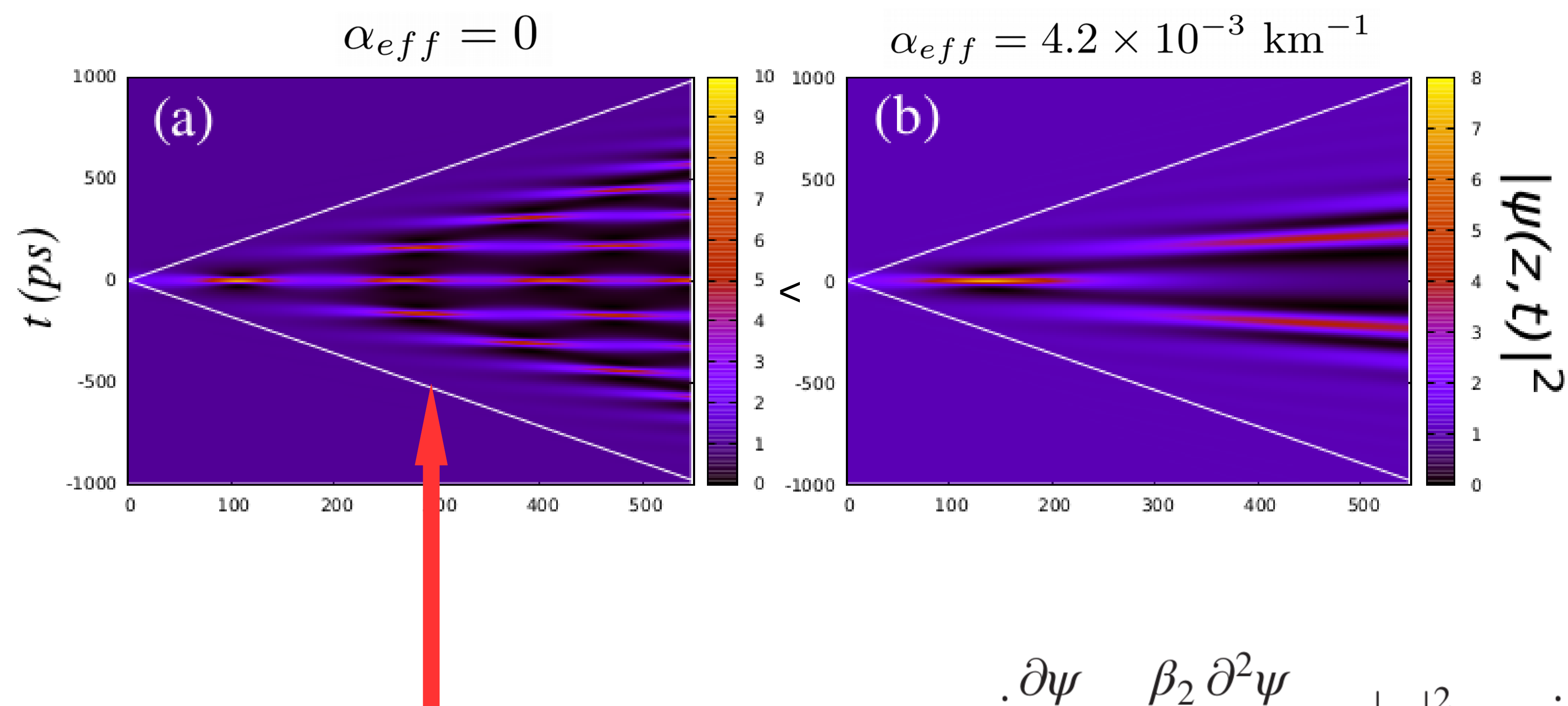


$$t_{\pm} = \pm 2\sqrt{2\beta_2 \gamma P_0 z}$$

$$i\frac{\partial \psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} - \gamma|\psi|^2\psi - i\frac{\alpha_{\text{eff}}}{2}\psi$$

A. E. Kraych, P. Suret, G. El and S. Randoux, "Nonlinear Evolution of the Locally Induced Modulational Instability in Fiber Optics" Phys. Rev. Lett. **122**, 054101 (2019)

Numerical simulations

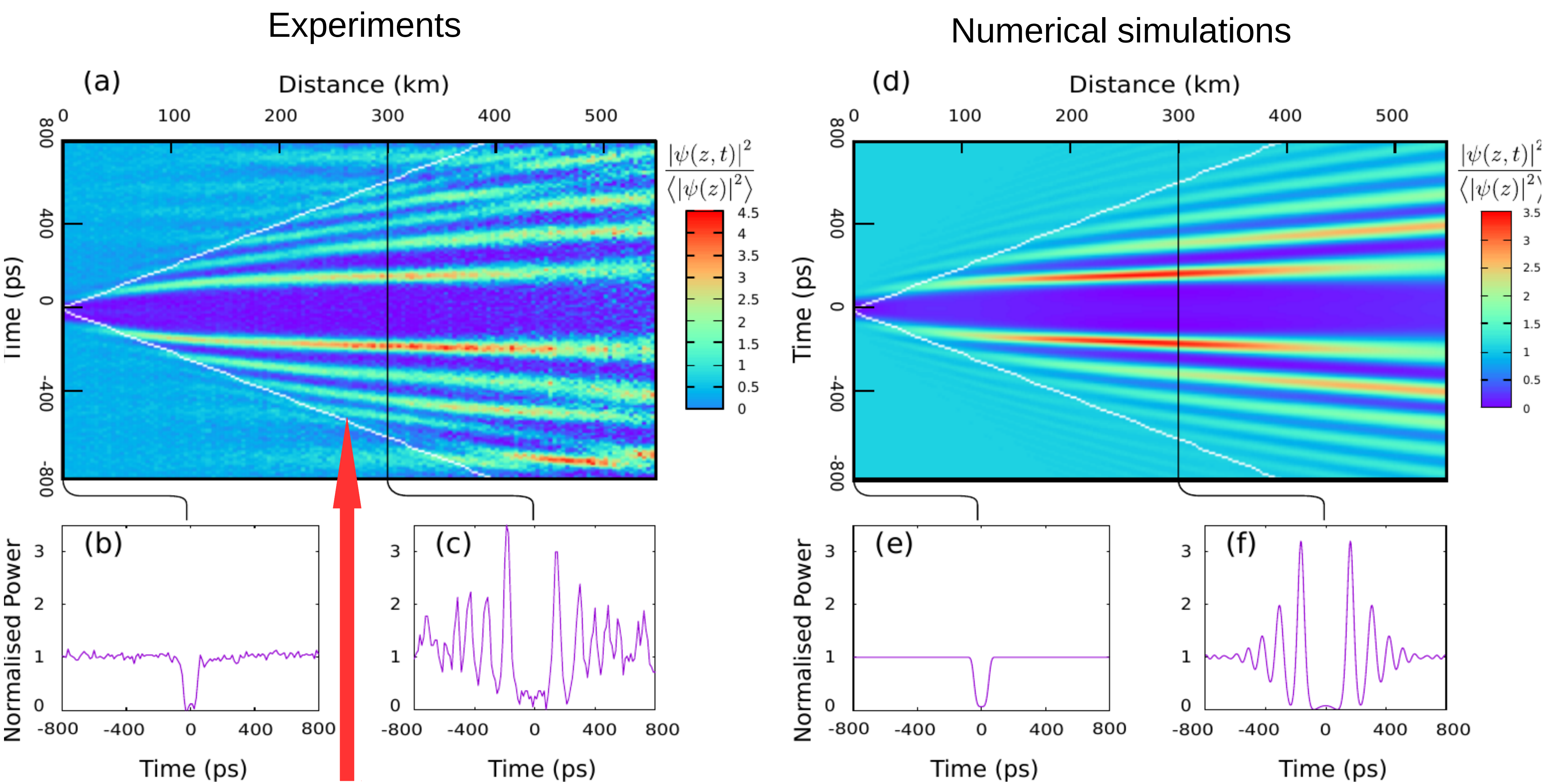


$$t_{\pm} = \pm 2\sqrt{2\beta_2 \gamma P_0} z$$

$$i\frac{\partial \psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} - \gamma|\psi|^2\psi - i\frac{\alpha_{\text{eff}}}{2}\psi$$

A. E. Kraych, P. Suret, G. El and S. Randoux, "Nonlinear Evolution of the Locally Induced Modulational Instability in Fiber Optics" Phys. Rev. Lett. **122**, 054101 (2019)

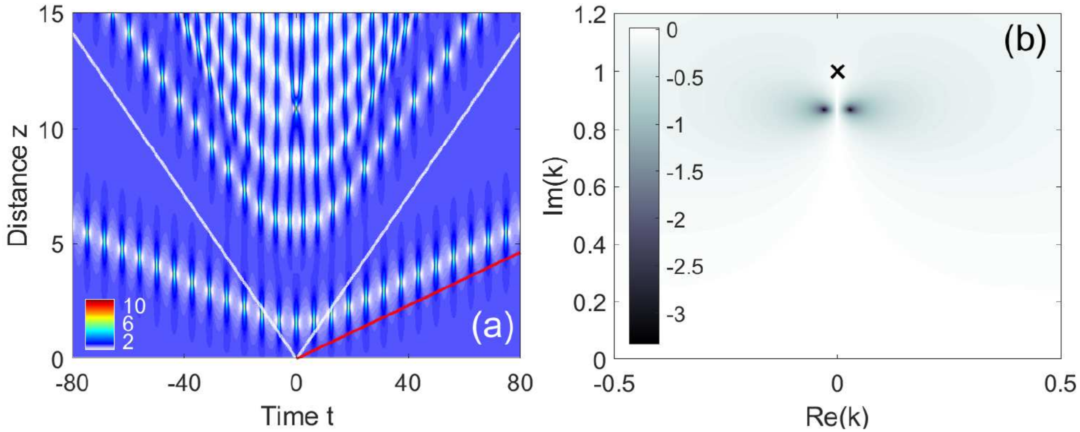
Localized perturbations



$$t_{\pm} = \pm 2\sqrt{2\beta_2 \gamma P_0} z$$

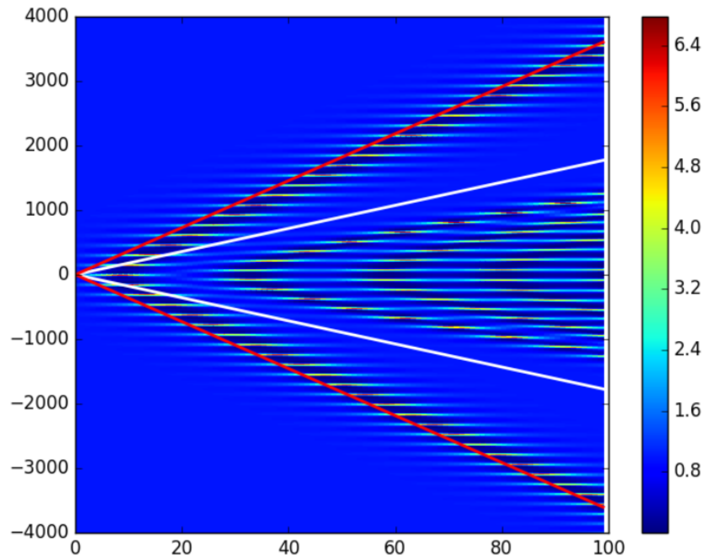
A. E. Kraych, P. Suret, G. El and S. Randoux, “Nonlinear Evolution of the Locally Induced Modulational Instability in Fiber Optics” Phys. Rev. Lett. **122**, 054101 (2019)

Localized perturbations

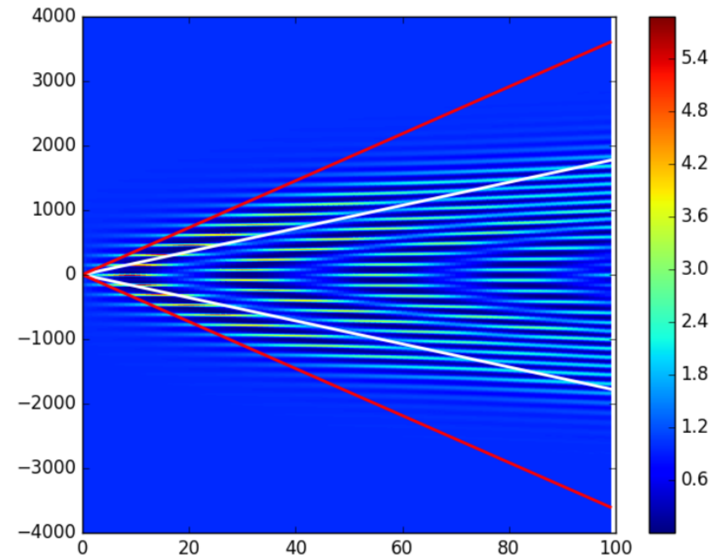


M. Conforti, S. Li, G. Biondini, and S. Trillo
"Auto-modulation versus breathers in the nonlinear stage of modulational instability",
Optics Letters Vol. 43, Issue 21, pp. 5291-5294 (2018)

Numerical simulations



$$\alpha_{eff} = 0$$

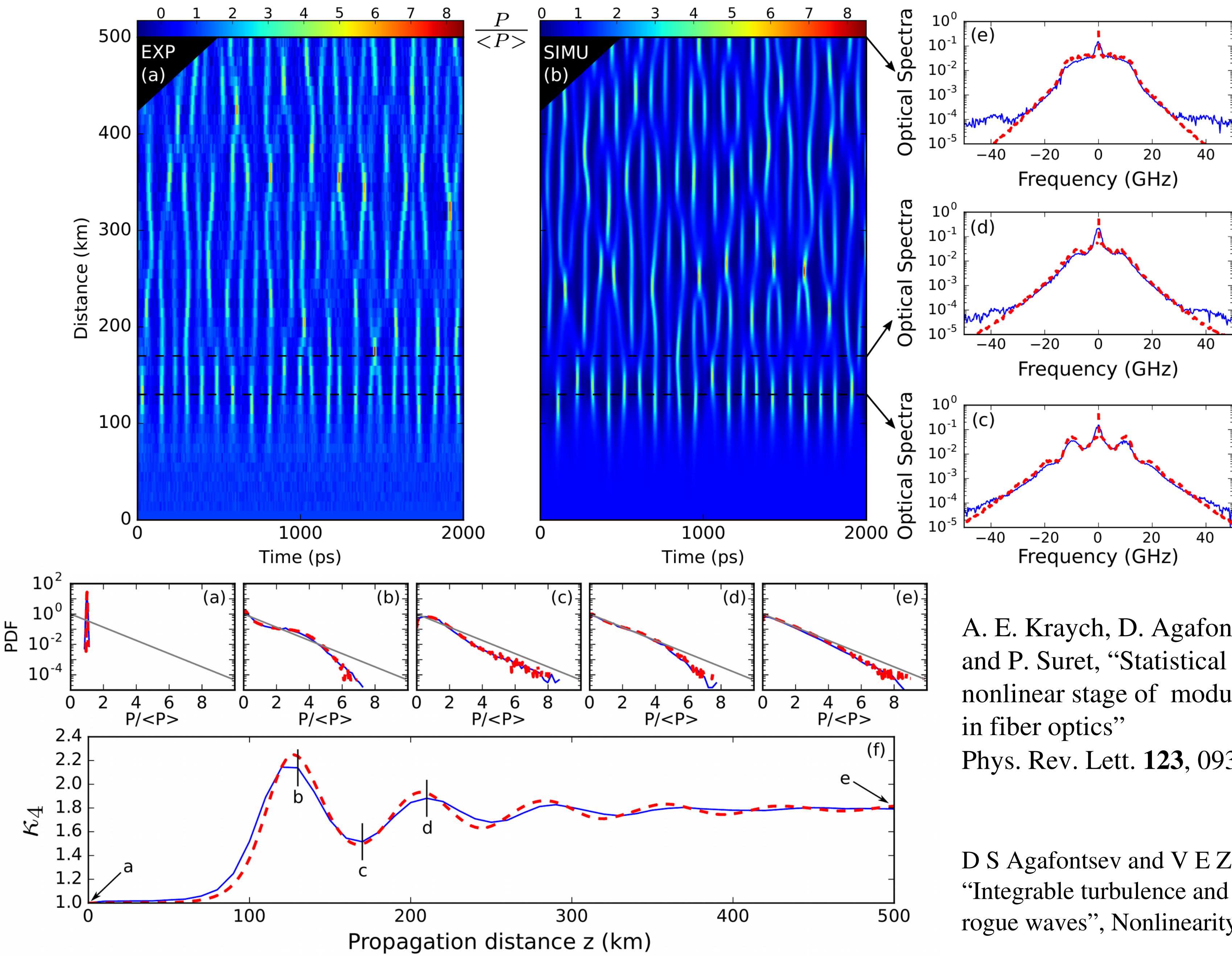


$$\alpha_{eff} = 4.2 \times 10^{-3}\ \mathrm{km}^{-1}$$

$$i\frac{\partial \psi}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 \psi}{\partial t^2} - \gamma |\psi|^2 \psi - i\frac{\alpha_{\mathrm{eff}}}{2}\psi$$

Optical fiber experiments

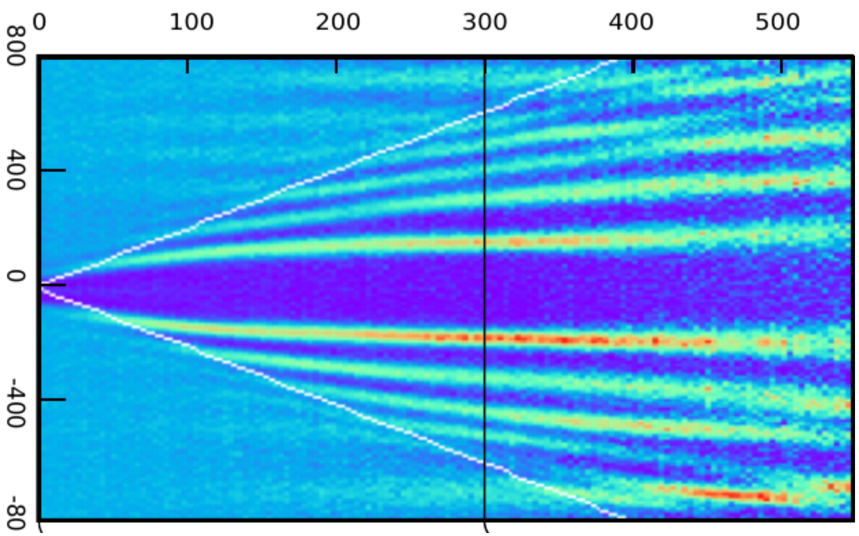
Without the localized pertubation : noise-driven modulational instability



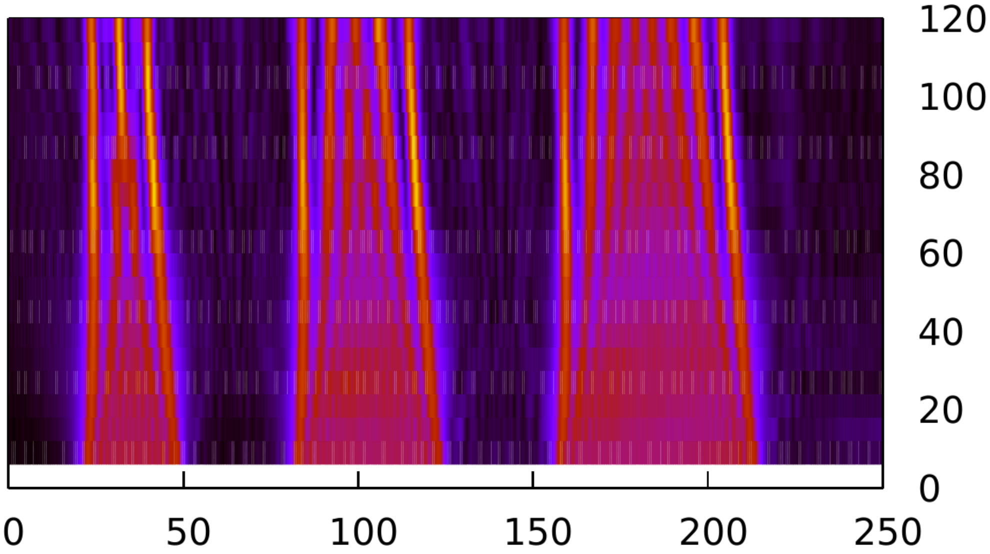
A. E. Kraych, D. Agafontsev, S. Randoux and P. Suret, "Statistical properties of nonlinear stage of modulation instability in fiber optics"
Phys. Rev. Lett. **123**, 093902 (2019)

D S Agafontsev and V E Zakharov, "Integrable turbulence and formation of rogue waves", Nonlinearity **28,** 2791 (2015)

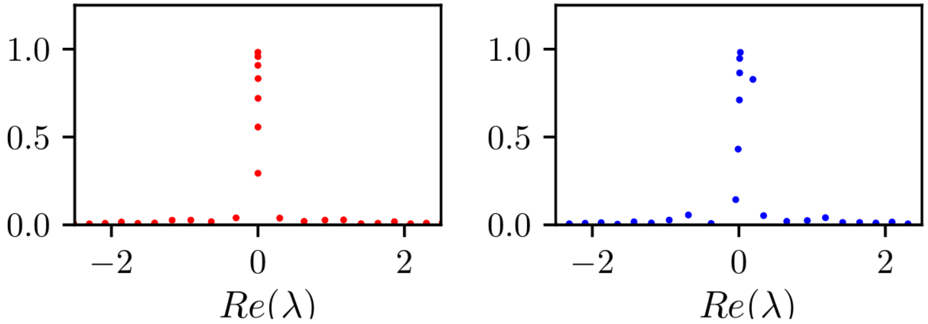
Optical fiber experiments



Water wave experiments



Nonlinear spectral analysis (numerical IST)



Nonlinear propagation of one-dimensional waves: some recent experimental results

Thank you for your attention!