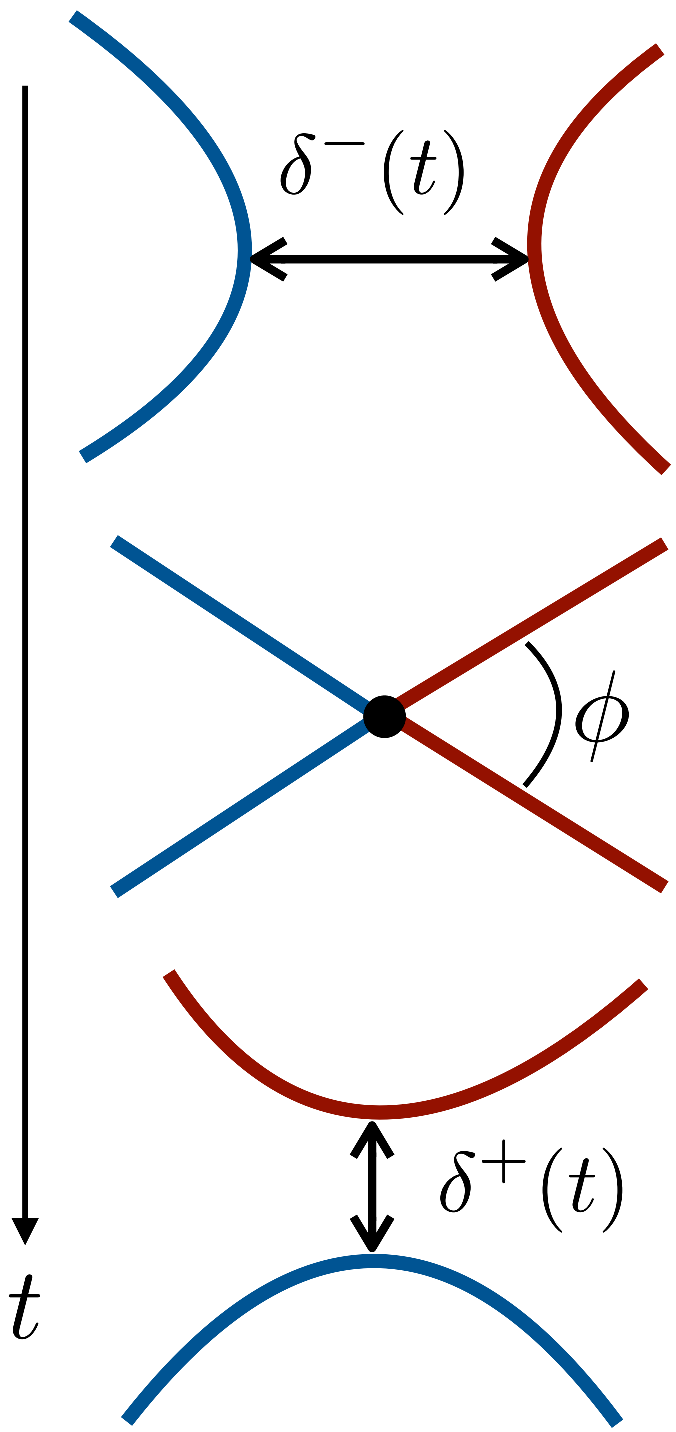
manuscript in preparation

IRREVERSIBLE DYNAMICS OF SUPERFLUID VORTEX RECONNECTIONS

DAVIDE PROMENT,
UNIVERSITY OF EAST ANGLIA (UK)

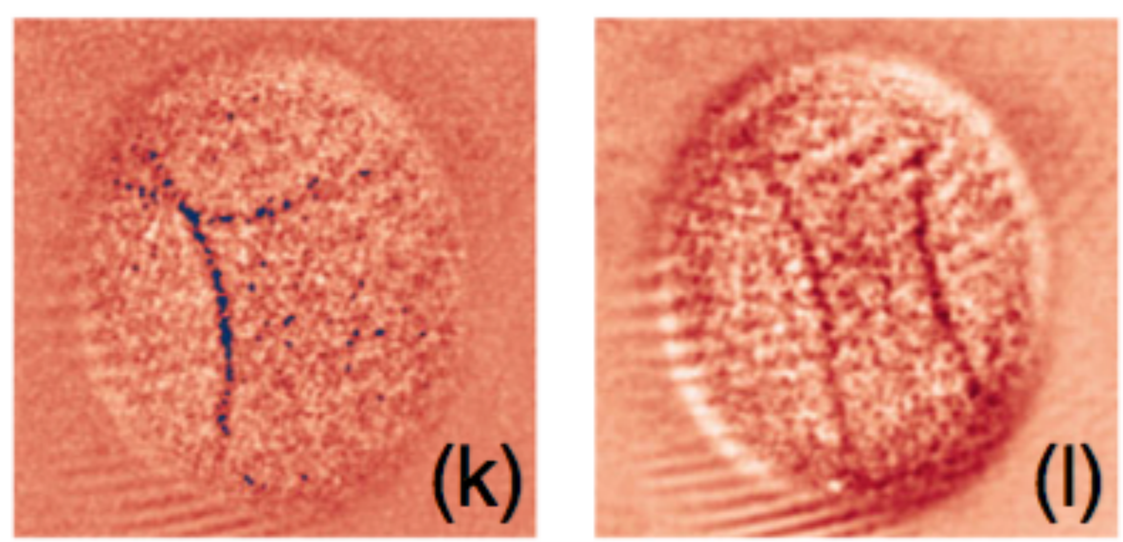
Joint work with: Alberto Villois and Giorgio Krstulovic

RECONNECTIONS IN SUPERFLUIDS



[Paoletti et al., PNAS 2008]

Vortex reconnections in superfluid liquid helium (top) and BEC of cold atoms (bottom)



[Serafini et al., PRL 2015]

THE GROSS-PITAEVSKII MODEL [Pitaevskii & Stringari, 2003]

$$\imath\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g|\psi|^2\psi=0$$

Madelung transformation $\psi=\sqrt{\rho}\exp(\imath\phi)$

$$\boxed{\mathbf{u}=\hbar/m\nabla\phi\,,\ \rho=m|\psi|^2}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{u})=0$$

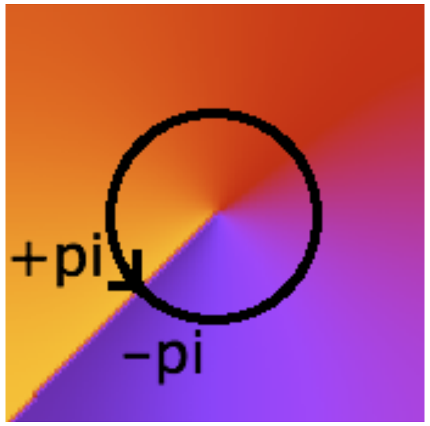
$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=\nabla\left[-\frac{g}{m}\rho+\frac{1}{m}V+\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right]$$

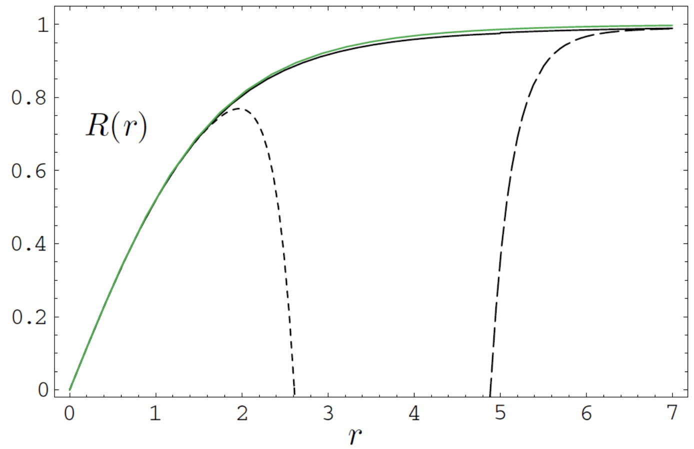
given bulk density ρ_0

$$\xi=\sqrt{\hbar^2/(2mg\rho_0)}$$

$$c=\sqrt{g\rho_0/m}$$

- inviscid, compressible, and irrotational fluid
- vortices are topological defects
- circulation quantised $\kappa=h/m$





TWO ENERGY FAMILIES IN THE GP MODEL

[Nore et al., PoF 1997]

- total energy is a constant of motion

$$E = \int |\nabla\psi|^2 dV + \frac{1}{2}\int \left(|\psi|^2 - \rho_0\right)^2 dV$$

$$E_{kin} = \frac{1}{4}\int \left(\sqrt{\rho}\,\mathbf{v}\right)^2 dV, \quad E_q = \frac{1}{4}\int \left(2\nabla\sqrt{\rho}\right)^2 dV, \quad E_{int} = \frac{1}{2}\int (\rho - \rho_0)^2 dV$$

Using Helmoltz's decomposition $\sqrt{\rho}\,\mathbf{v} = \left(\sqrt{\rho}\,\mathbf{v}\right)^i + \left(\sqrt{\rho}\,\mathbf{v}\right)^c$

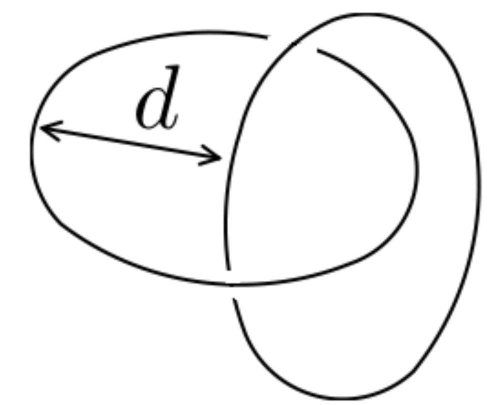
density perturbations: $E^c_{kin} = \frac{1}{4}\int \left[\left(\sqrt{\rho}\,\mathbf{v}\right)^c\right]^2 dV$

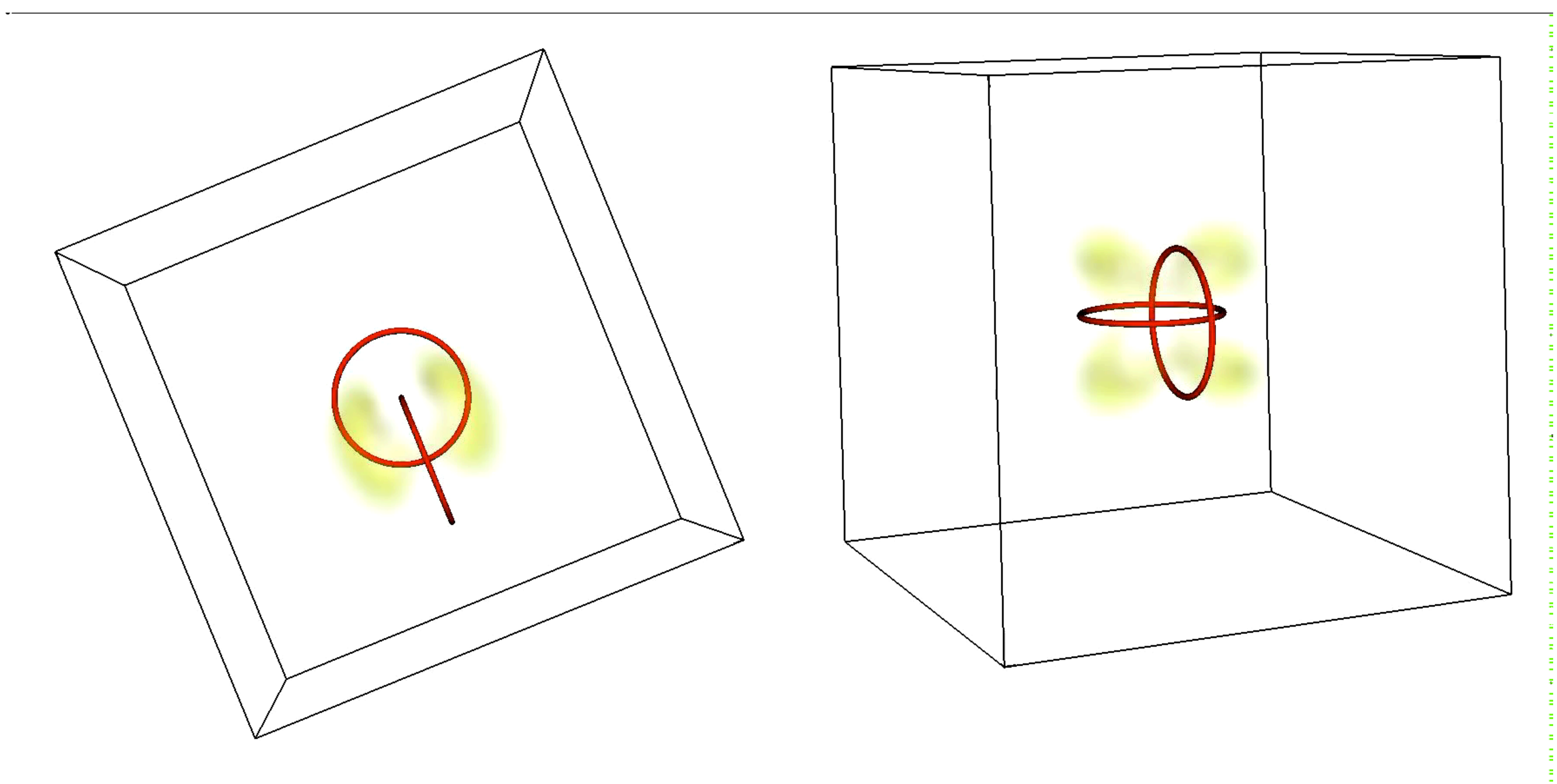
quantised vortices: $E^i_{kin} = \frac{1}{4}\int \left[\left(\sqrt{\rho}\,\mathbf{v}\right)^i\right]^2 dV$

- energy transfers between vortices and sound families

OUR NUMERICAL EXPERIMENTS ON RECONNECTIONS

- decay of two linked rings
- vary the offset d, spanning over 49 different configurations
- track vortex filaments and measure sound

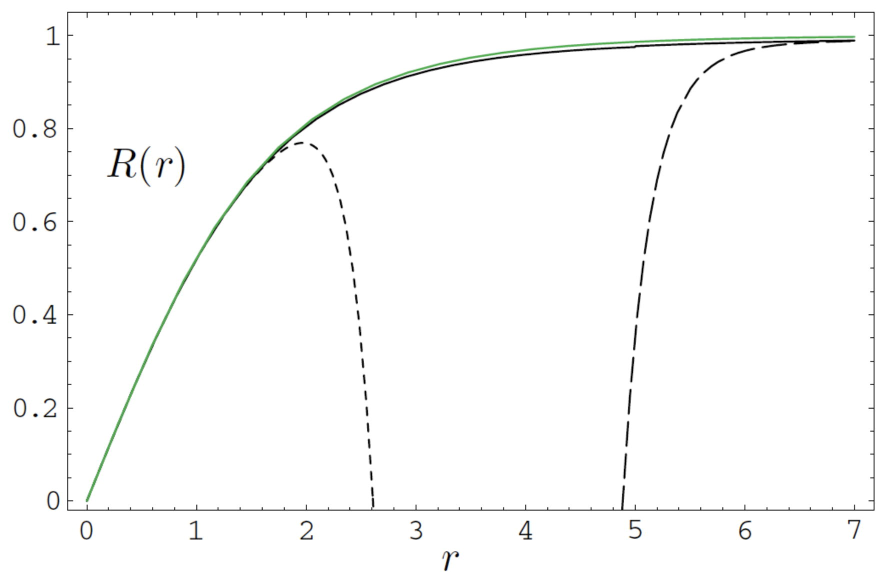




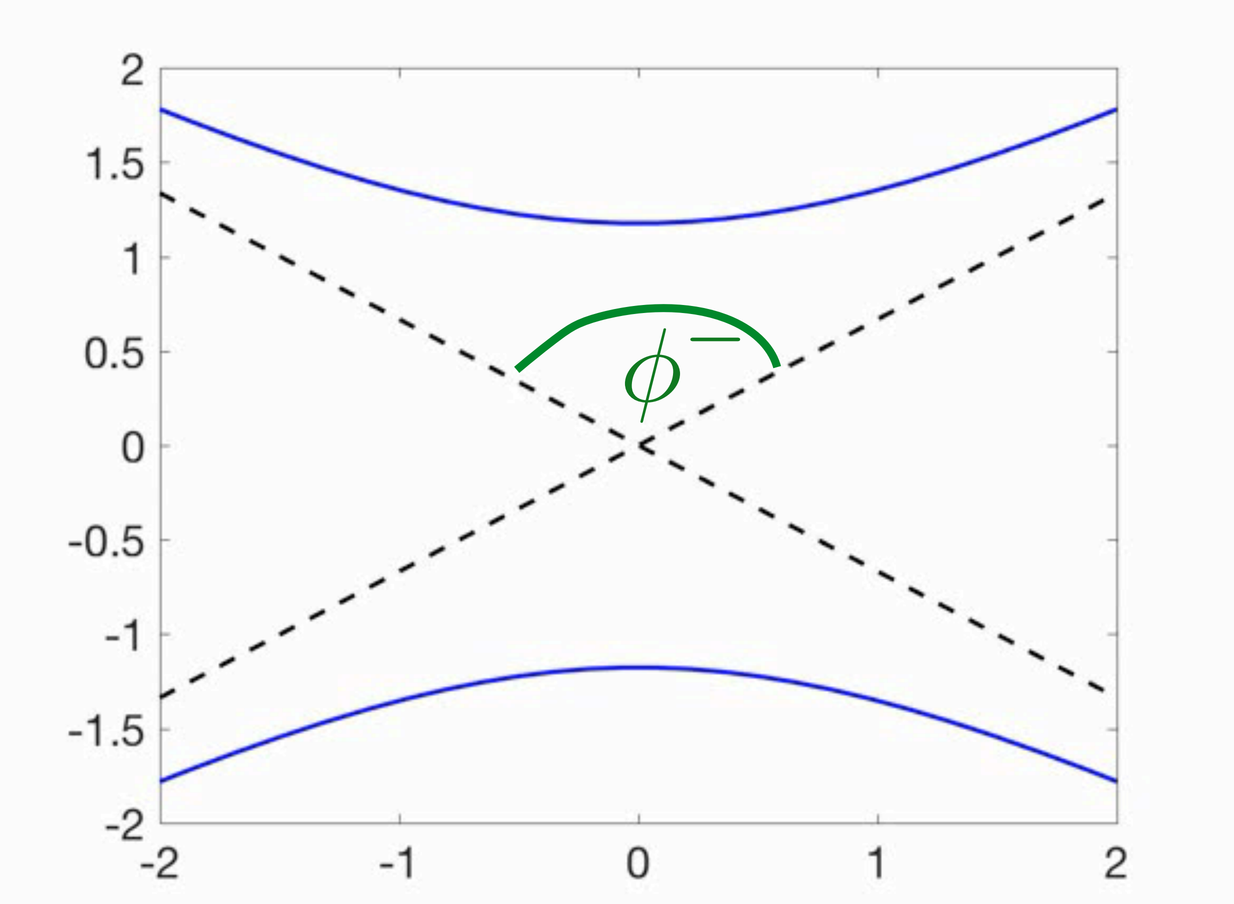
ABOUT RECONNECTION: LINEAR THEORY APPROXIMATION

[Nazarenko & West, JLTP 2003]

$$\delta^{\pm}(t) \leq \xi \quad \Longrightarrow \quad \imath\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi - \cancel{g|\psi|^2}\psi = 0$$

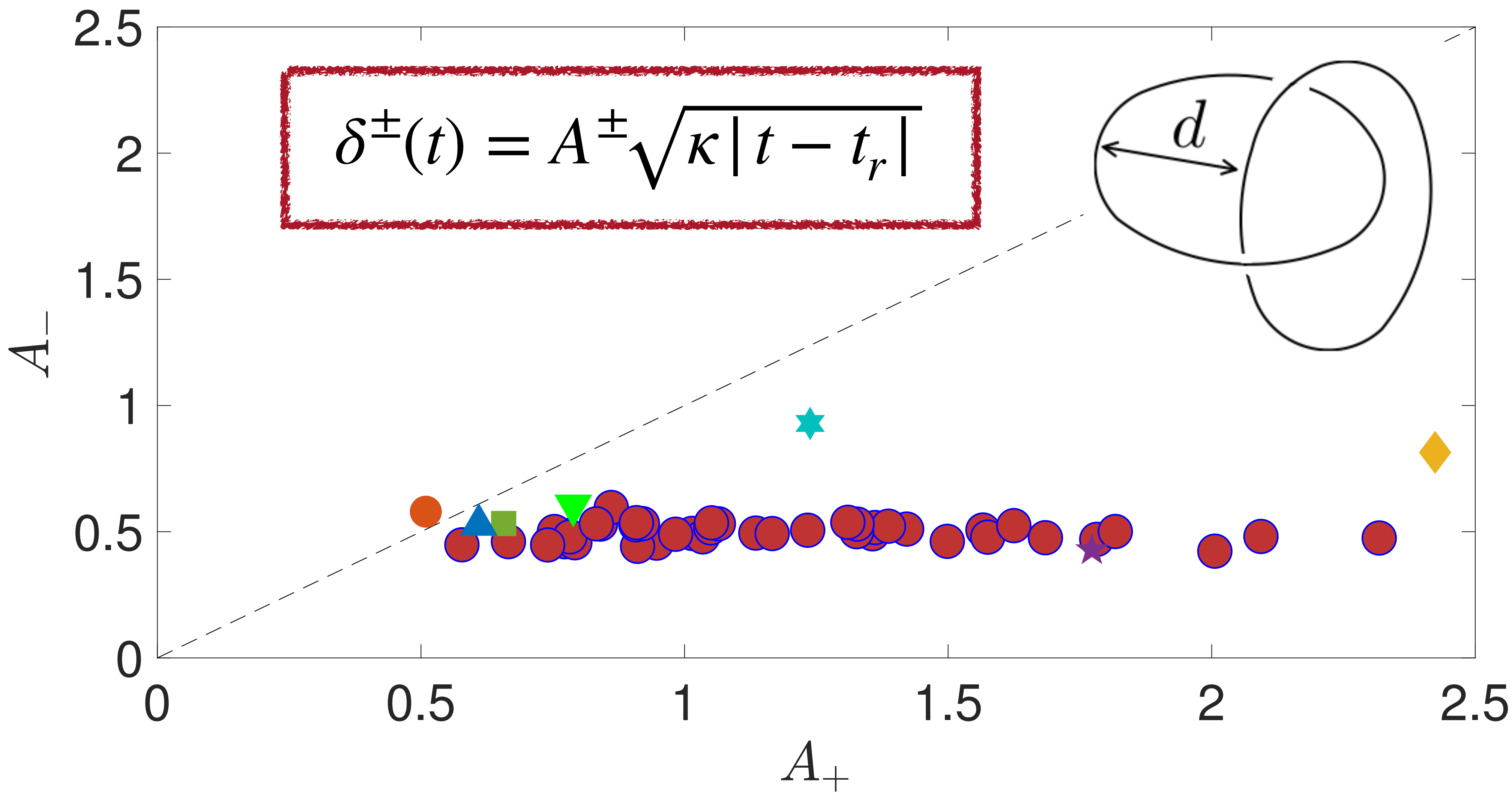


$$\boxed{\begin{aligned}\delta^{\pm}(t) &= A^{\pm}\sqrt{\kappa|t-t_r|}\\ \phi^{-} &= 2\arctan(A^{+}/A^{-})\end{aligned}}$$



- same scaling $\delta \propto t^{1/2}$ before and after, only the pre-factors $A^{\pm}$ change
- filaments follow locally the branches of an hyperbola

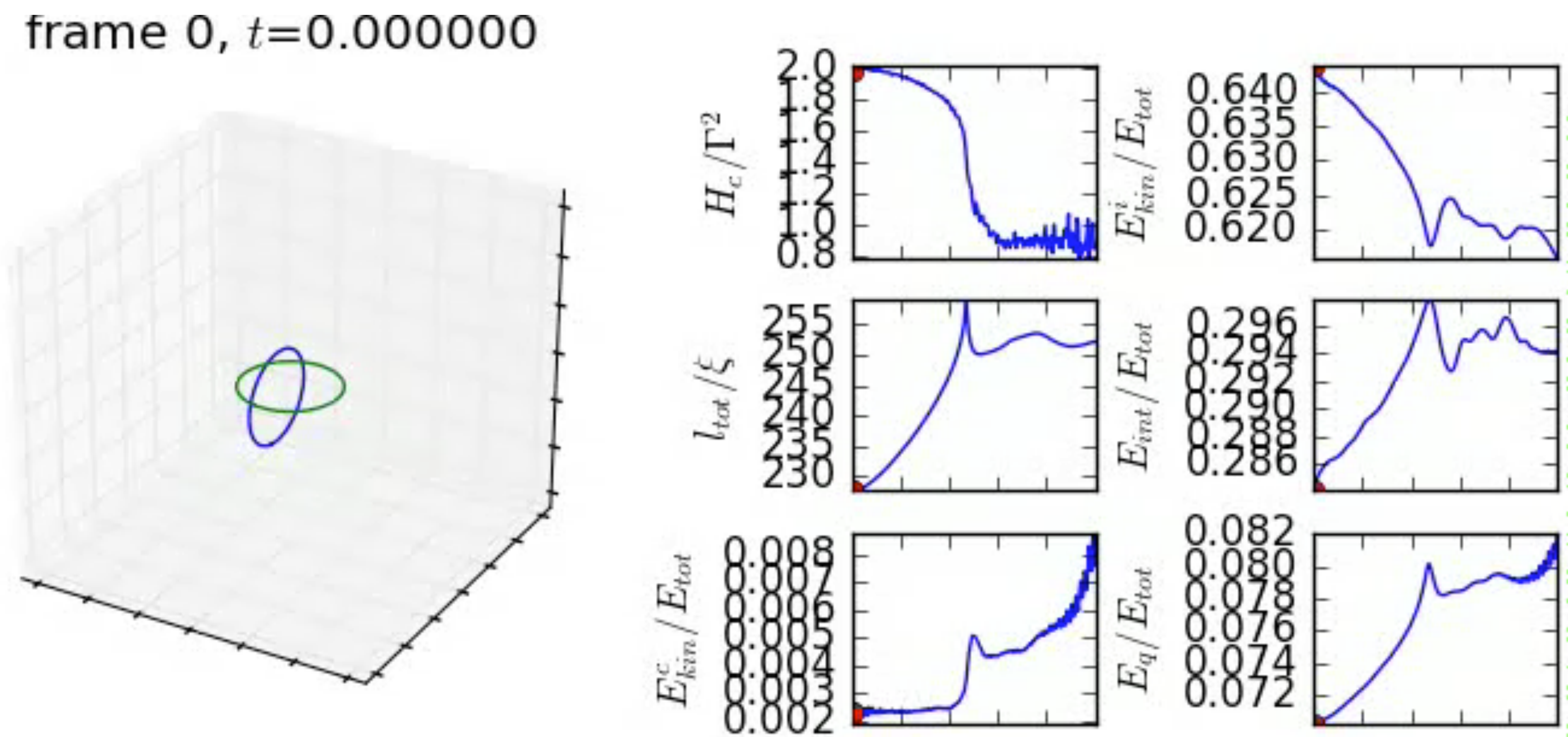
ASYMMETRY IN RATES OF APPROACH AND SEPARATION



Red circles correspond to data of the present work, all other symbols are from [Villois et al., PRFluids 2018]

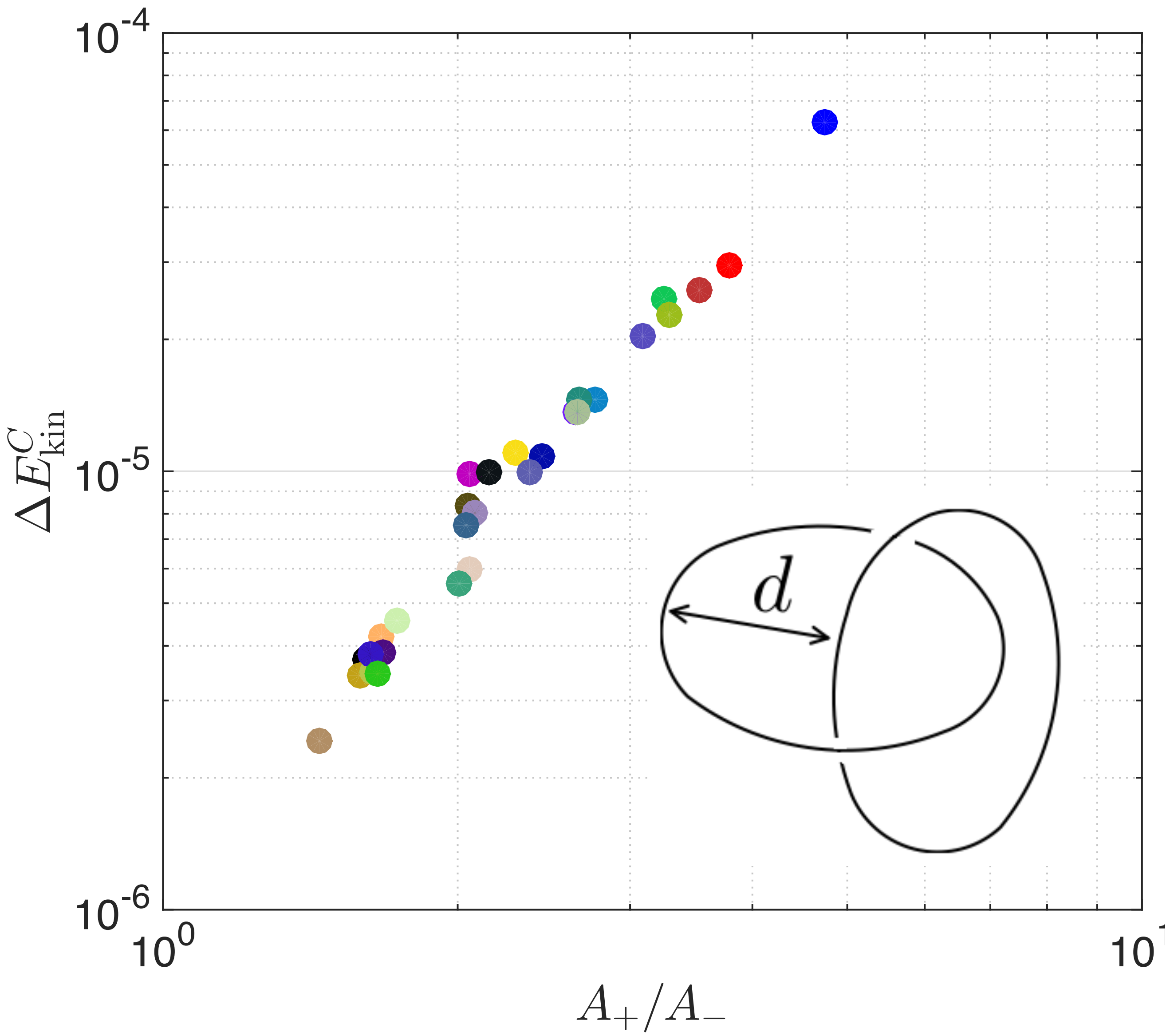
HOW TO EXPLAIN THIS ASYMMETRY?

A TYPICAL EVOLUTION

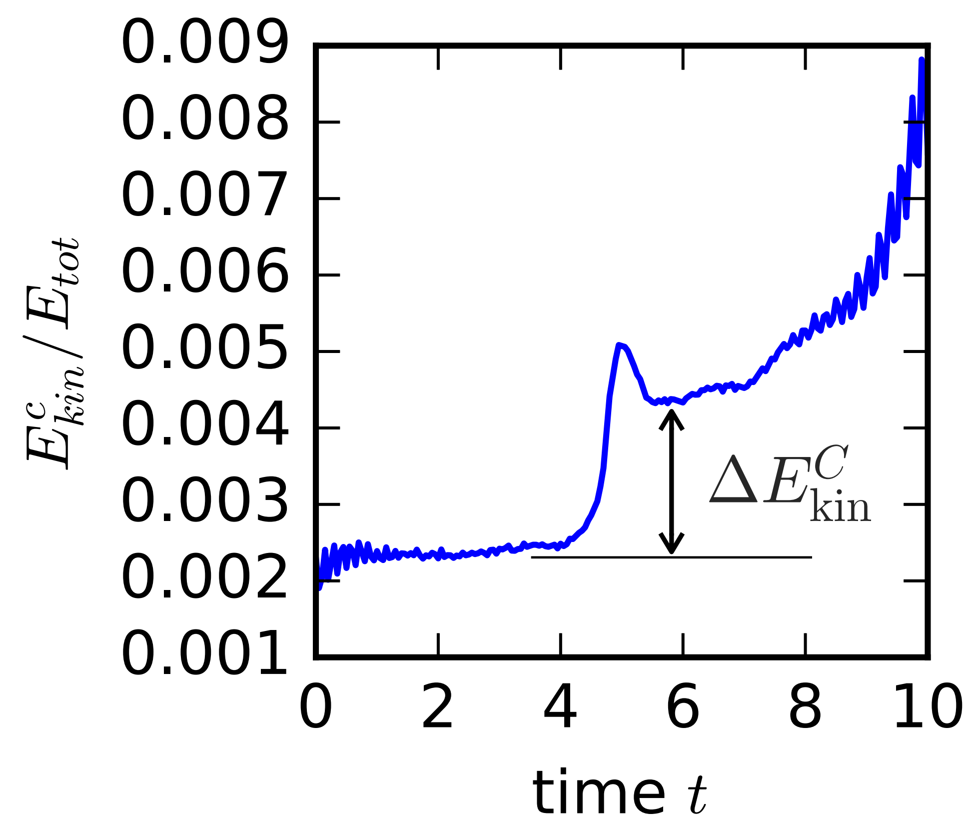


Evolution of various measurable quantities during the reconnection, including energy components

COMPRESSIBLE KINETIC ENERGY GROWTH



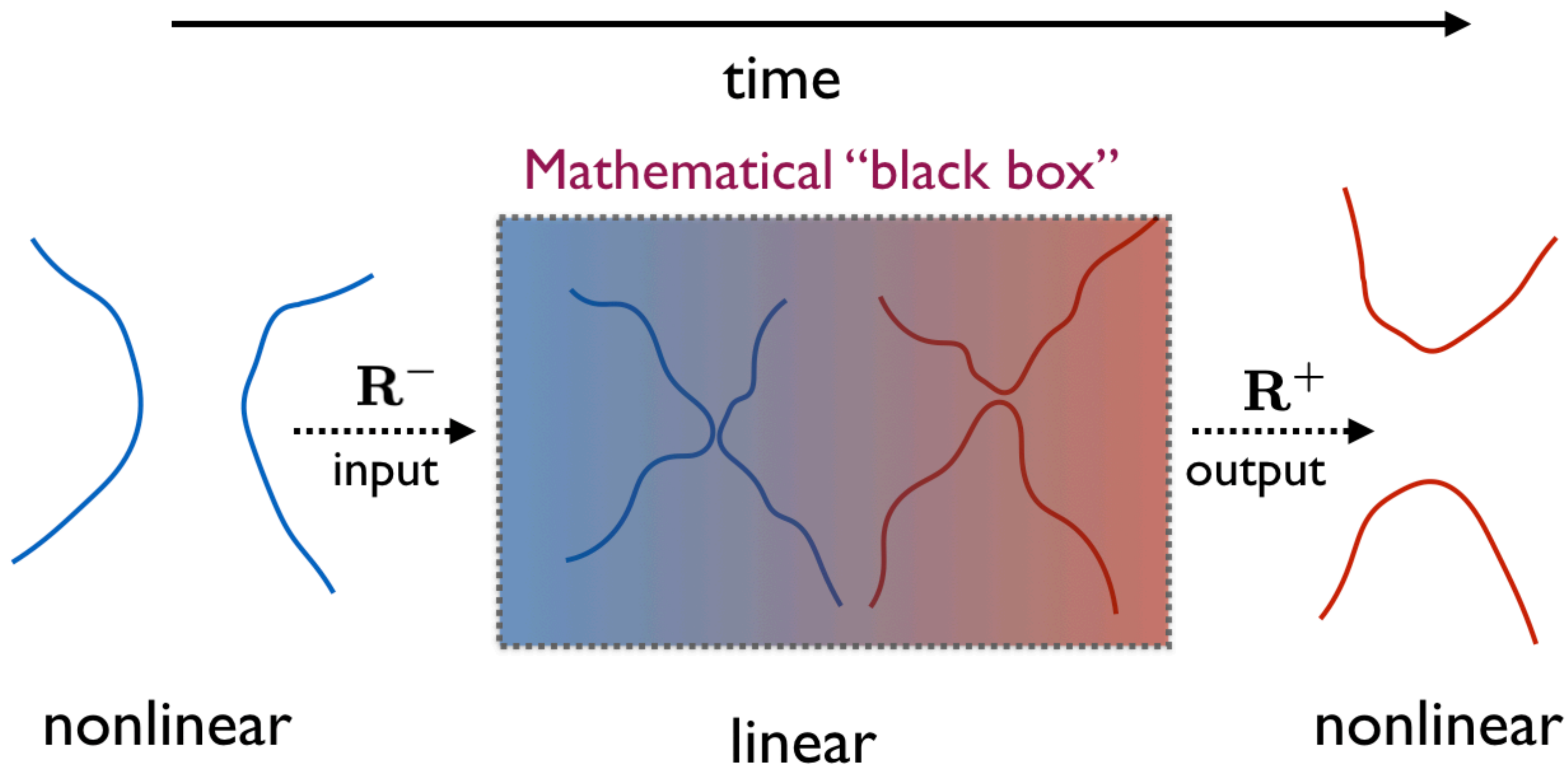
Example of energy growth during a reconnection



Growth of the compressible kinetic energy during the reconnection vs. the ratio A^+/A^- for the 49 different realisations

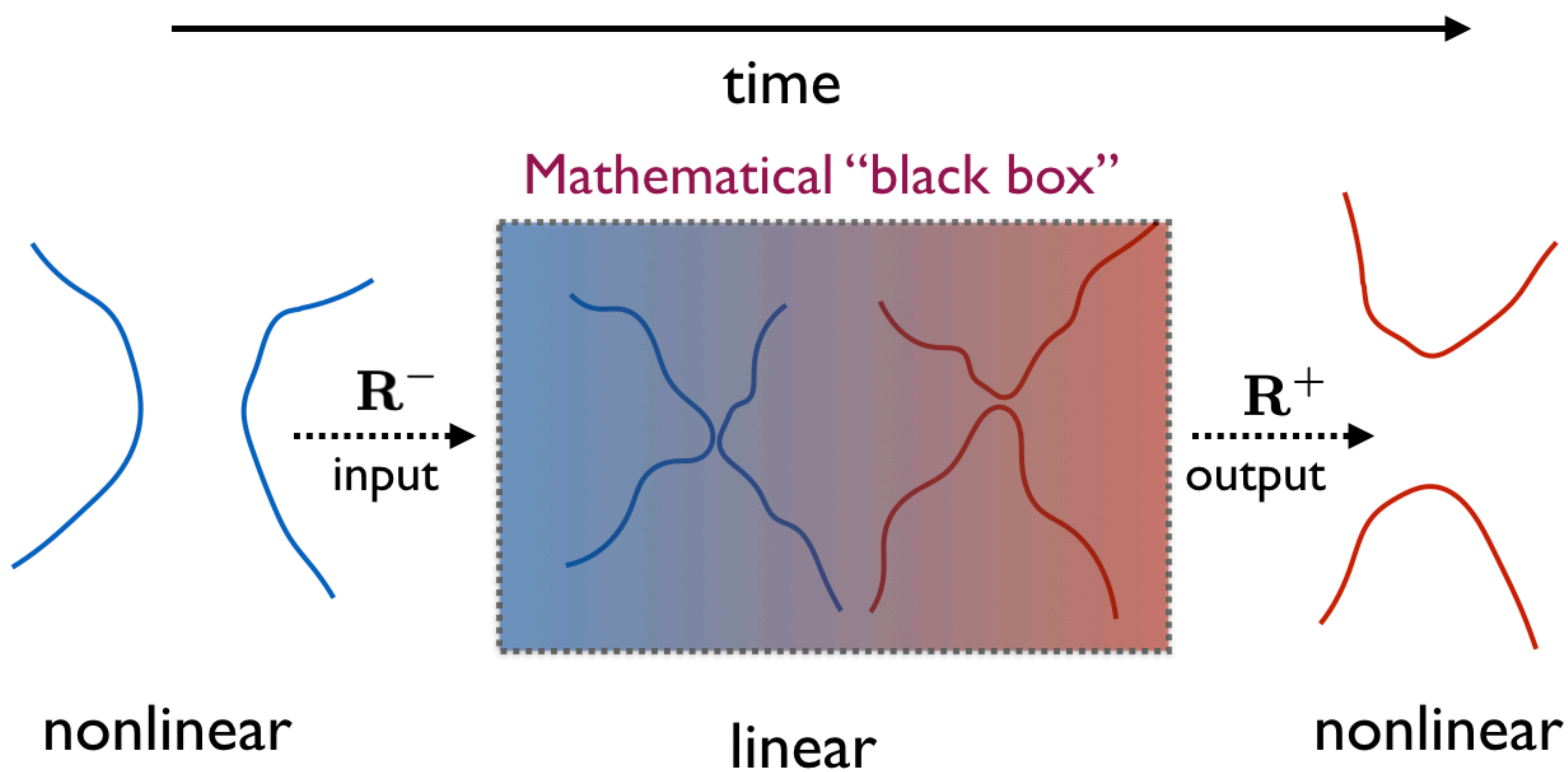
HOW TO PREDICT THIS BEHAVIOUR?

A PHENOMENOLOGICAL MATCHING THEORY



- when $\delta(t) \geq \delta_{\mathrm{lin}}$ nonlinear theory using vortex filament model or local induction approximation (LIA)
- when $\delta(t) \leq \delta_{\mathrm{lin}}$ linear theory as described before
- matching of the two theories at $\delta(t) = \delta_{\mathrm{lin}}$

A PHENOMENOLOGICAL MATCHING THEORY



from nonlinear theory

momentum: $\mathbf{P}_{\text{fil}}^{\pm} = \dfrac{\kappa}{2} \oint \mathbf{R}^{\pm} \times d\mathbf{R}^{\pm}$

energy: $E_{\text{fil}}^{\pm} \propto |\kappa|^2 \oint |d\mathbf{R}^{\pm}|$

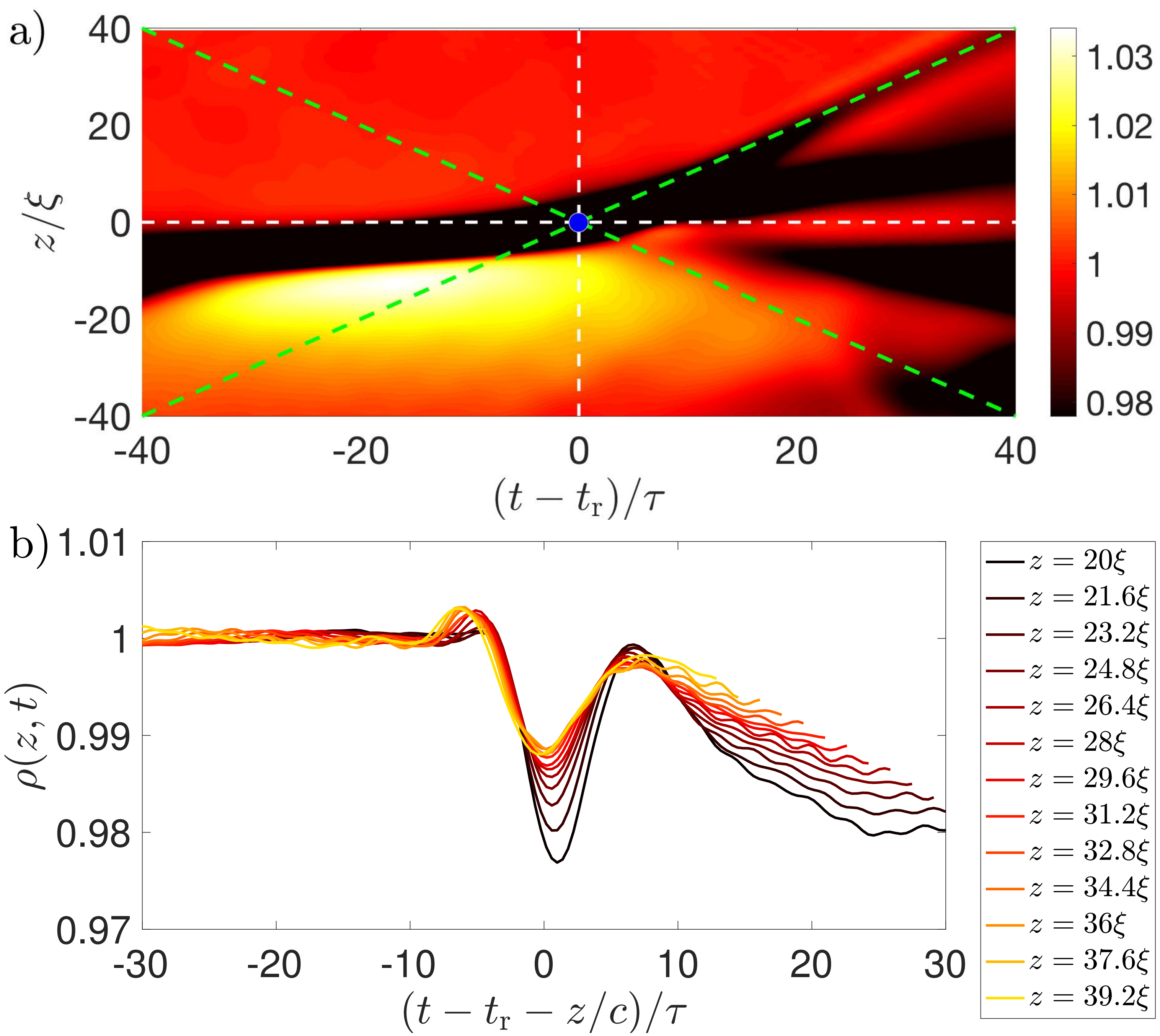
$\Longrightarrow$

$$\boxed{\begin{aligned} \Delta \mathbf{P}_{\text{fil}} &= \mathbf{P}_{\text{fil}}^{+} - \mathbf{P}_{\text{fil}}^{-} \\ \Delta E_{\text{fil}} &= E_{\text{fil}}^{+} - E_{\text{fil}}^{-} \end{aligned}}$$

[Pismen, 1999]

CONVERSION OF FILAMENT'S MOMENTUM INTO SOUND

$$\mathbf{\Delta P}_{\text{wav}} = -\,\mathbf{\Delta P}_{\text{fil}} \propto \left(0,\ 0,\ \frac{1 + A^+/A^-}{\sqrt{A^+/A^-}}\right) \implies \Delta P_{\text{wav,z}} > 0$$



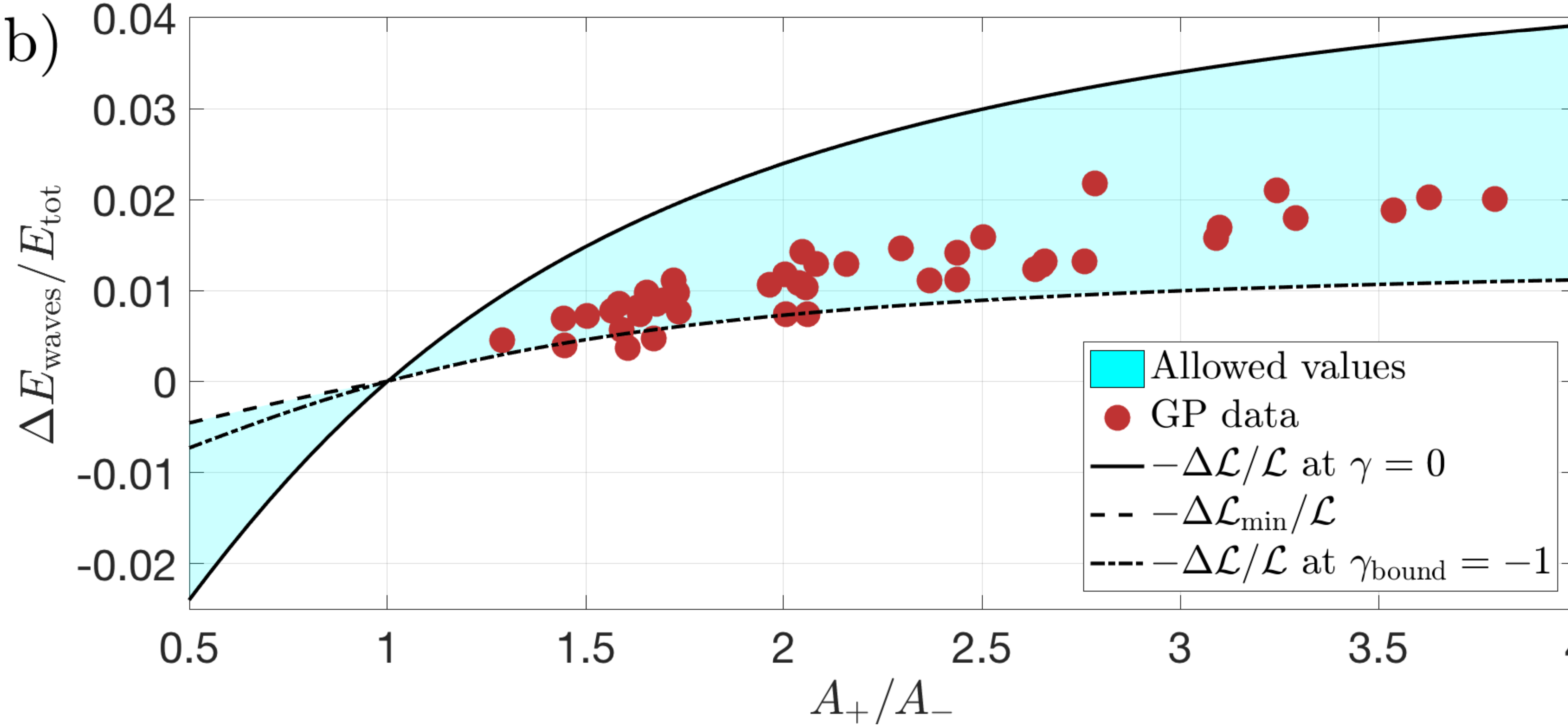
Example of sound pulse emission propagating orthogonally to the reconnection plane

- propagation at almost speed of sound (dashed green lines)
- some dispersion
- reduction in the sound minimum

$$\propto \frac{1}{(t - t_r - z/c)^2}$$

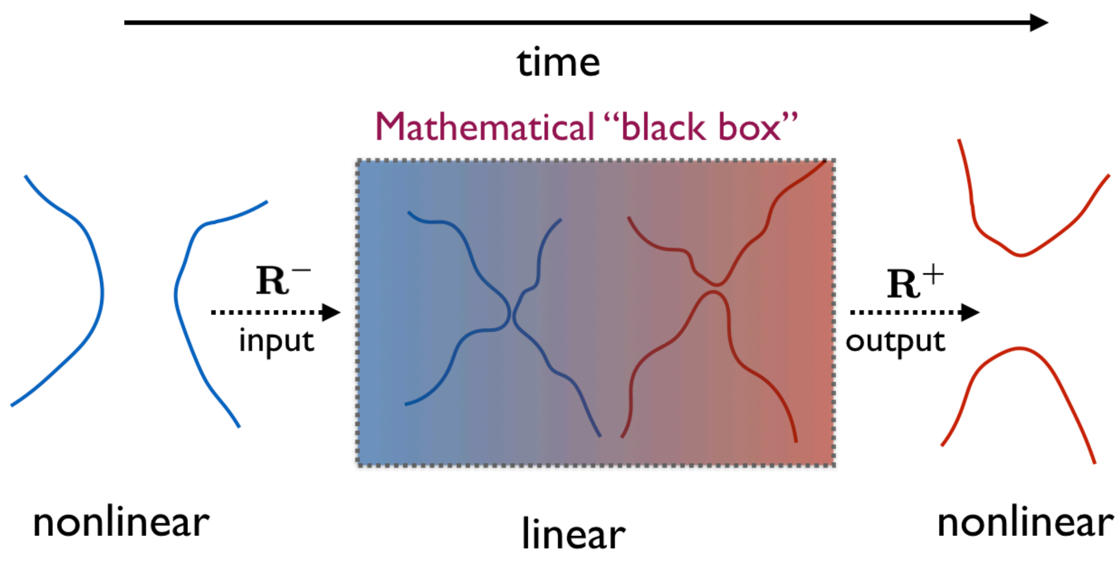
CONVERSION OF FILAMENT'S ENERGY INTO SOUND

$$\Delta E_{\mathrm{wav}} = -\Delta E_{\mathrm{fil}} \propto \Delta \mathscr{L} / \mathscr{L} \quad \text{(using LIA)}$$



- a range of values are allowed, when considering that reconnecting filaments do not lie exactly on a plane
- γ is a measure of the concavity in the z direction

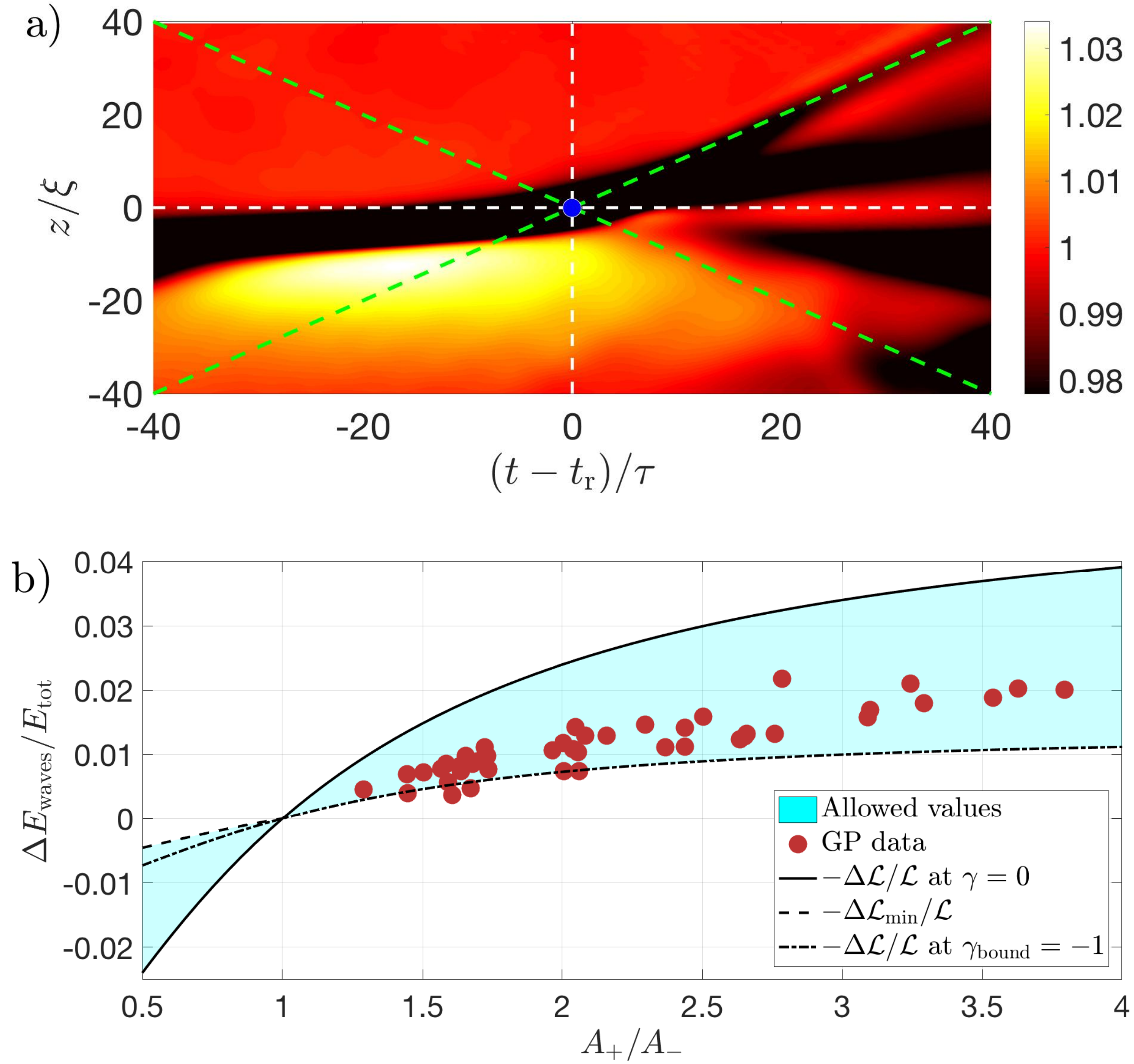
CONCLUSIONS



- linear momentum always lost in the negative direction (orthogonal to the reconnection plane), sound pulse has positive momentum
- $A^+ \geq A^-$ because it is energetically favourable
- energy radiated depends on the reconnecting angle

$$\phi^- = 2\arctan(A^+/A^-)$$

- phenomenological matching between linear and nonlinear theory



manuscript in preparation

THANKS FOR YOUR ATTENTION!

Joint work with: Alberto Villois and Giorgio Krstulovic

Acknowledgments

G.K., D.P. and A.V. were supported by the cost-share Royal Society International Exchanges Scheme (IE150527) in conjunction with CNRS. A.V. and D.P. were supported by the EPSRC First Grant scheme (EP/P023770/1). D.P. acknowledges the Fédération Doeblin for the support while visiting Nice in November 2017.