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SOUND EMISSION AND IRREVERSIBLE DYNAMICS DURING VORTEX RECONNECTIONS IN QUANTUM FLUIDS

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SOUND EMISSION AND IRREVERSIBLE DYNAMICS DURING VORTEX RECONNECTIONS IN QUANTUM FLUIDS

- Introduction on quantum fluids (superfluids)
- What are vortex reconnections?
- Evidence of irreversible dynamics
- Matching theory to explain this behaviour

WHAT IS A QUANTUM FLUID (SUPERFLUID)?

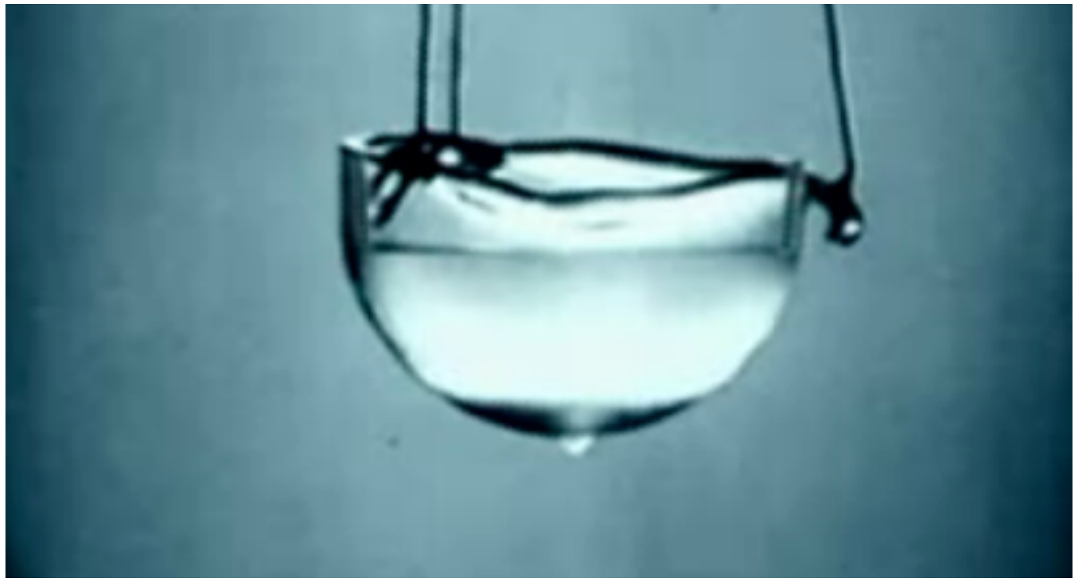
Mathematically (fluid mechanics)

- Total absence of viscosity
- Irrotational flow, but vortices exist as topological defects
- Vorticity is delta-supported and circulation is quantised (take only multiple values of the quantum of circulation

Physically (quantum mechanics, statistical mechanics, condensed matter)

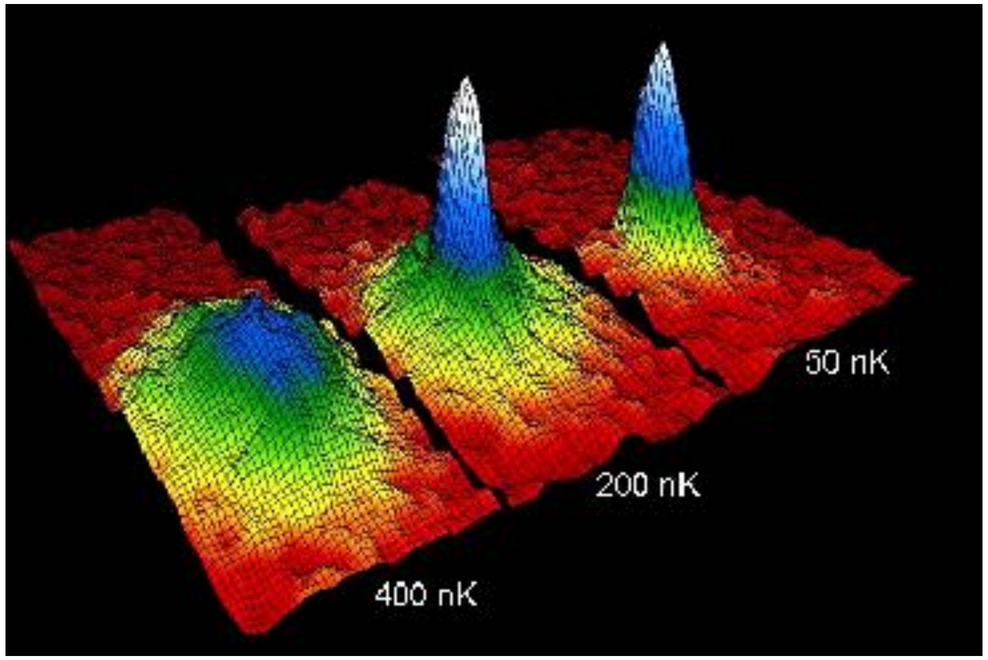
- Quantum fluids manifest at very low temperatures or at very high density
- Superfluidity is related to Bose-Einstein condensation
- Emergence of an order parameter that describes the system

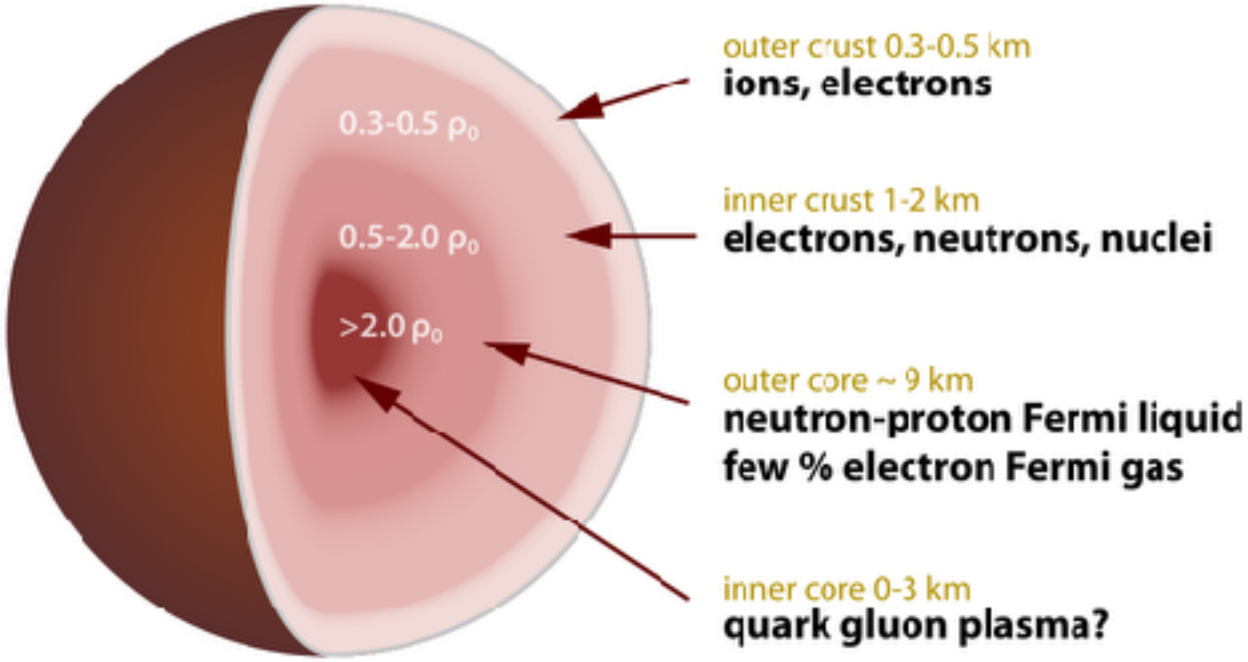
EXAMPLES OF QUANTUM FLUIDS



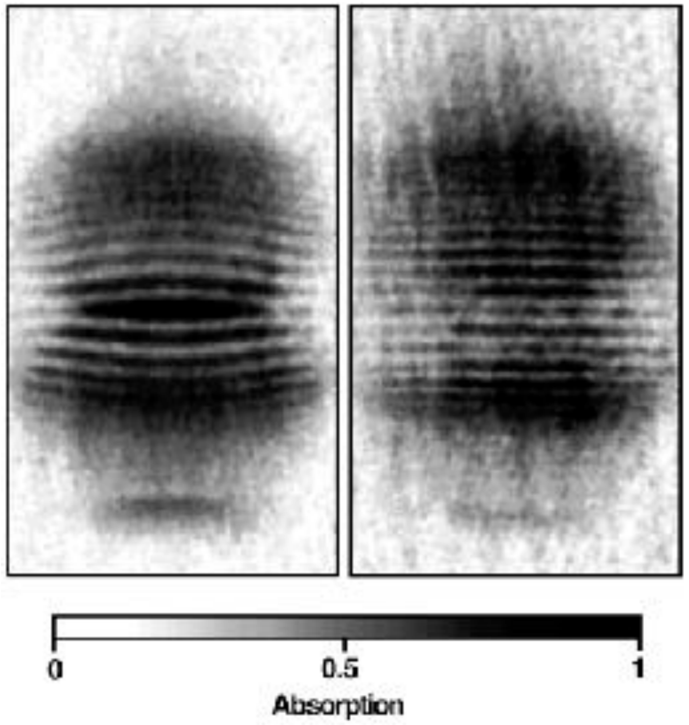
Superfluid liquid helium [Public Domain, Wikipedia]

Bose-Einstein condensates
[top: JILA group, bottom: Ketterle et al.]





Neutron stars [Robert Schulze, Wikipedia]



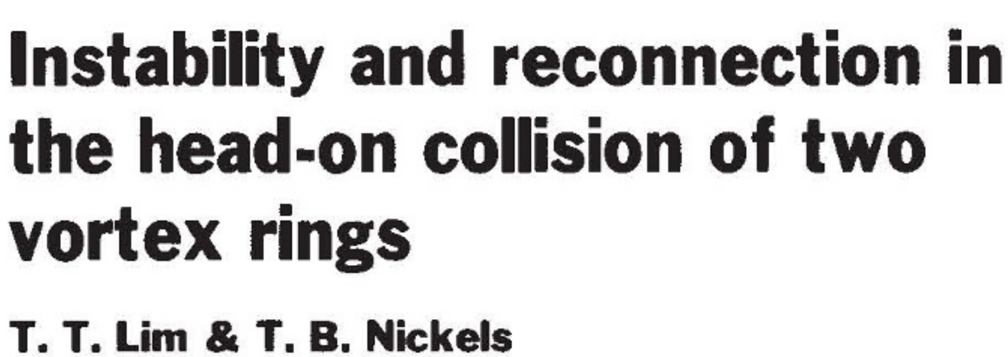
VORTEX RECONNECTION IN CLASSICAL FLUIDS

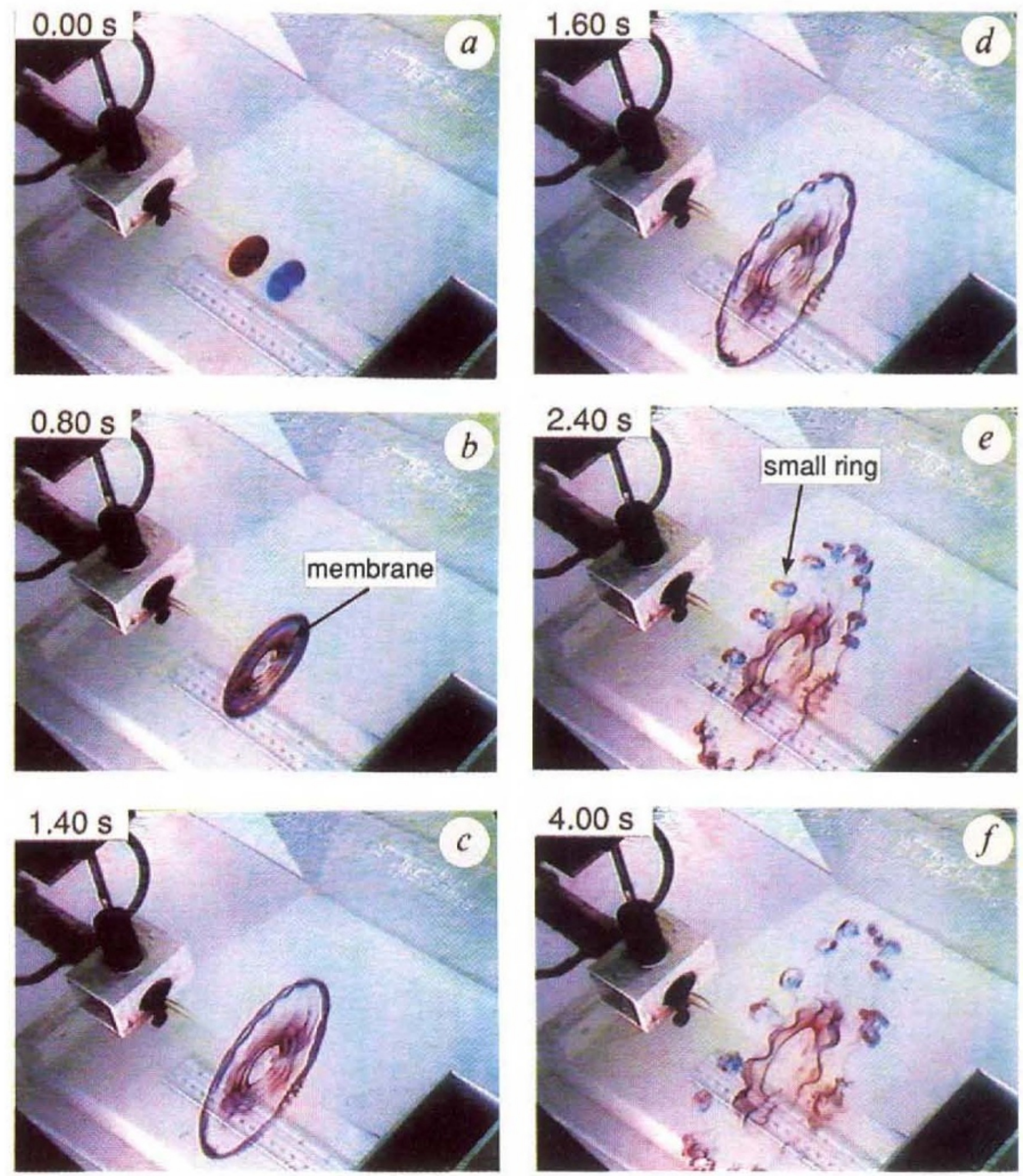
Before the reconnection

- Two vortex tubes (intense vorticity) approaching each others

After the reconnection

- Vortex tubes and other vortex structures emerge and separate





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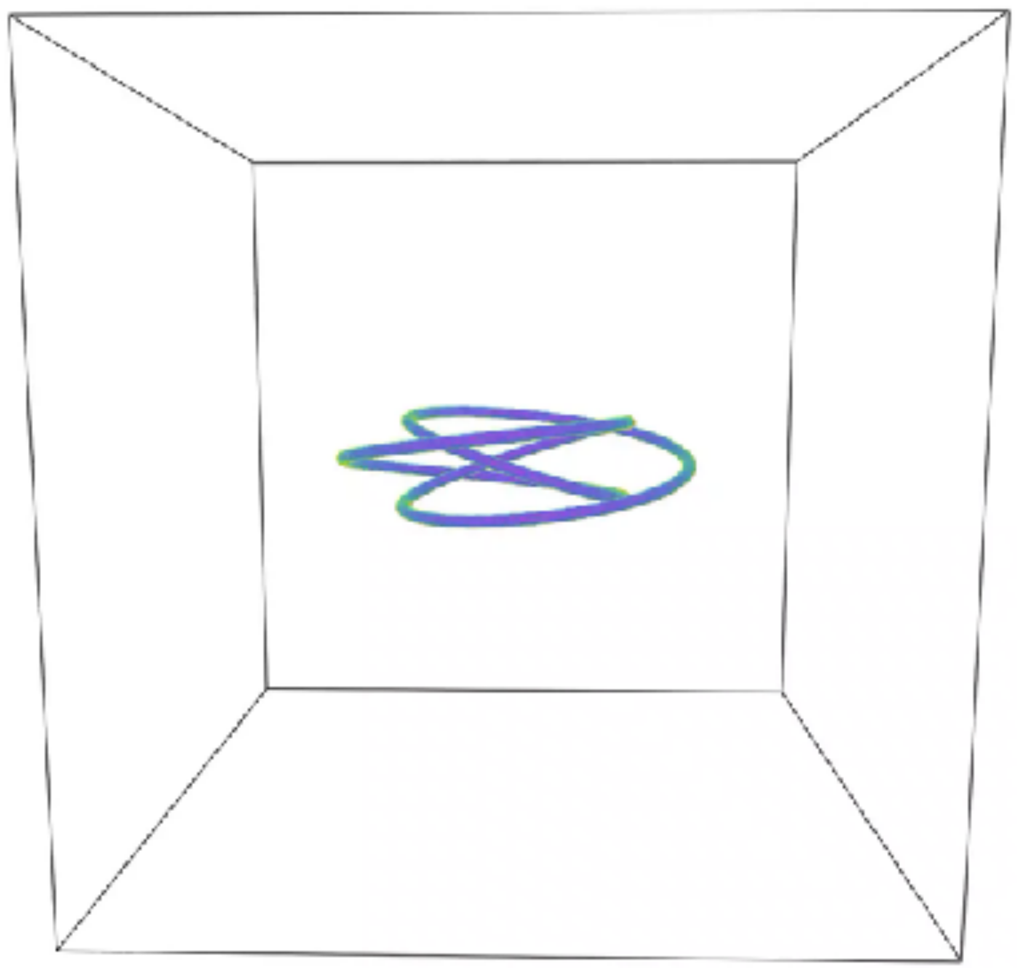
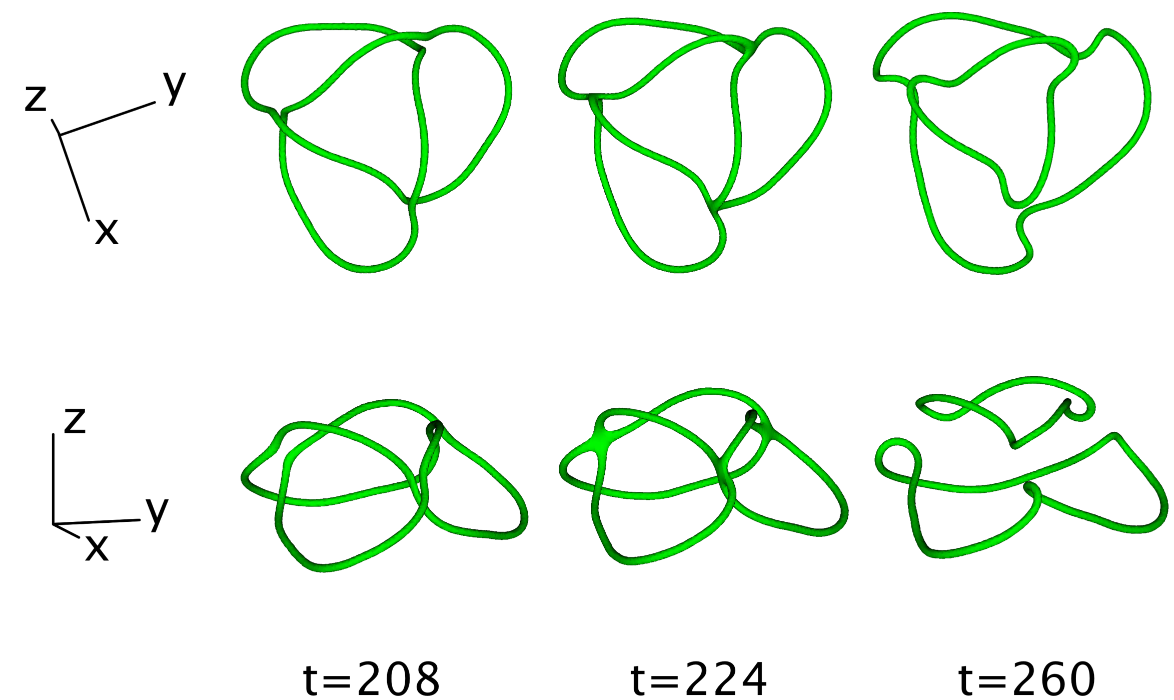
VORTEX RECONNECTION IN CLASSICAL FLUIDS

[Kleckner & Irvine, Nature 2013]

VORTEX RECONNECTION CLASSICAL VS. QUANTUM FLUIDS



Trefoil decaying in classical viscous fluids
[Kurstulovic, private communication]

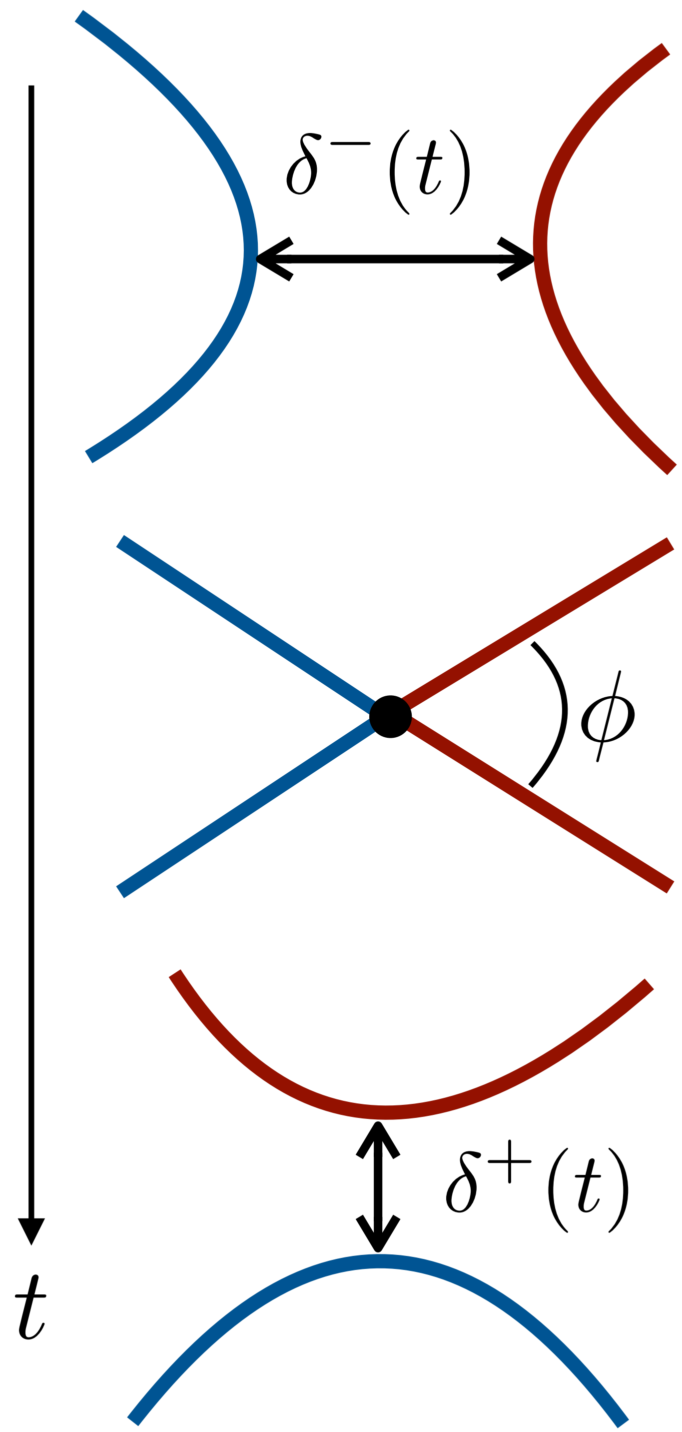


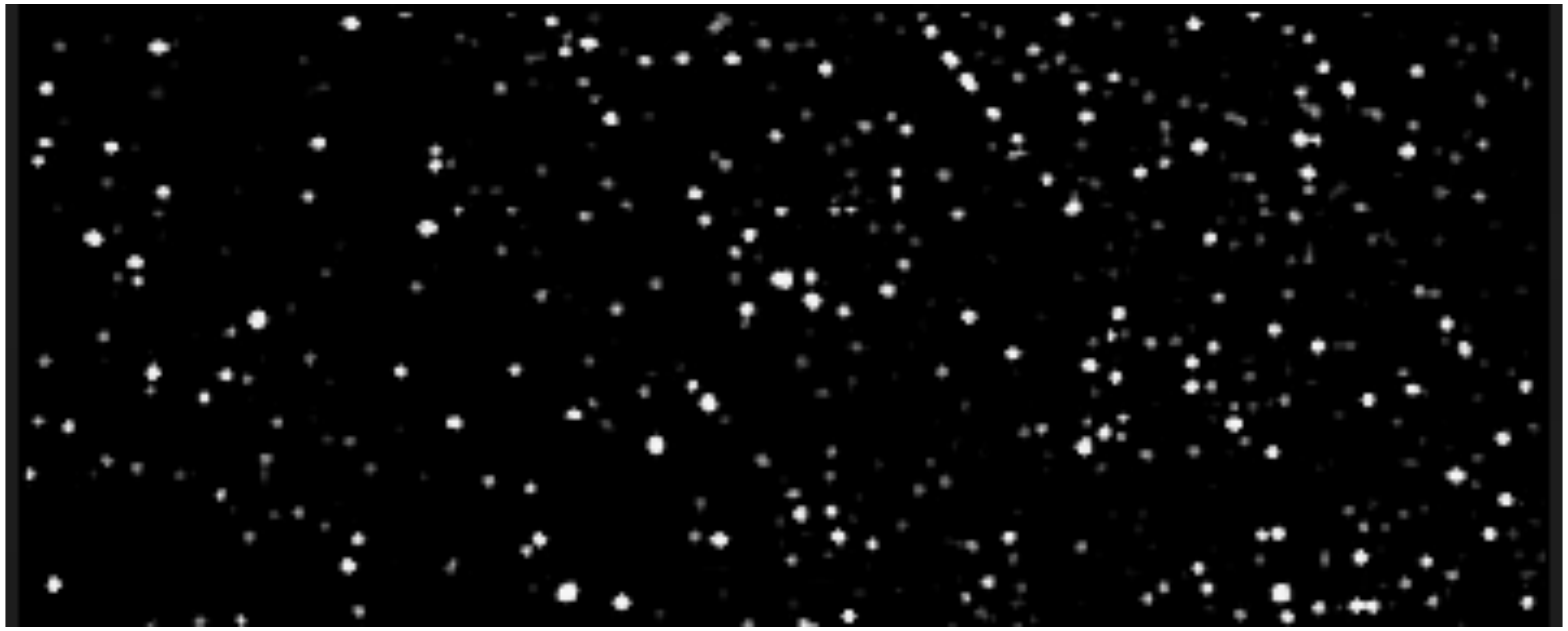
Trefoil decaying in classical quantum fluids
[Proment et al., PRE 2012]

- Complicate vortex structures are created after the reconnection

- As the circulation takes only quantised value, vortices simply reconnect exchanging their segments

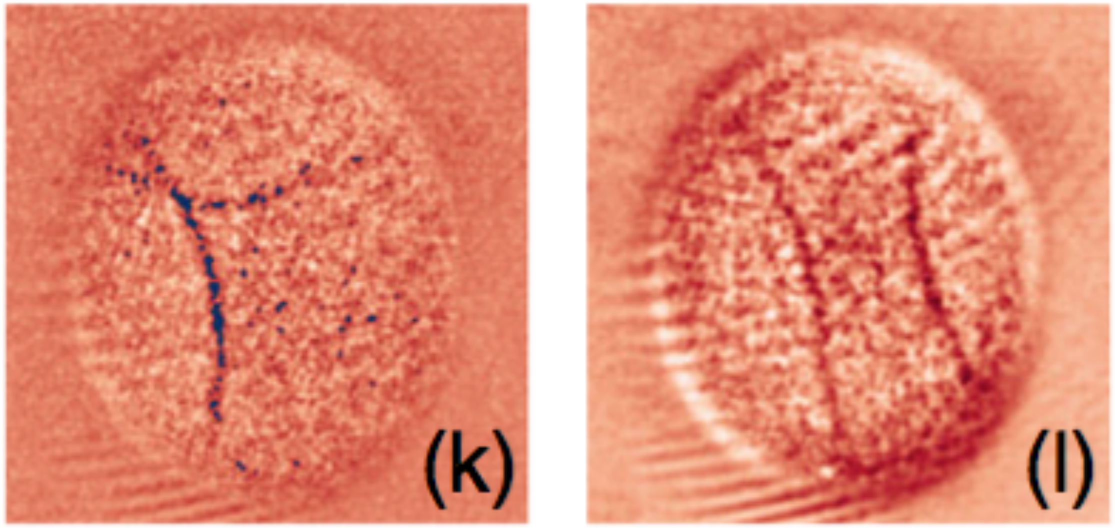
VORTEX RECONNECTIONS IN SUPERFLUIDS





[Paoletti et al., PNAS 2008]

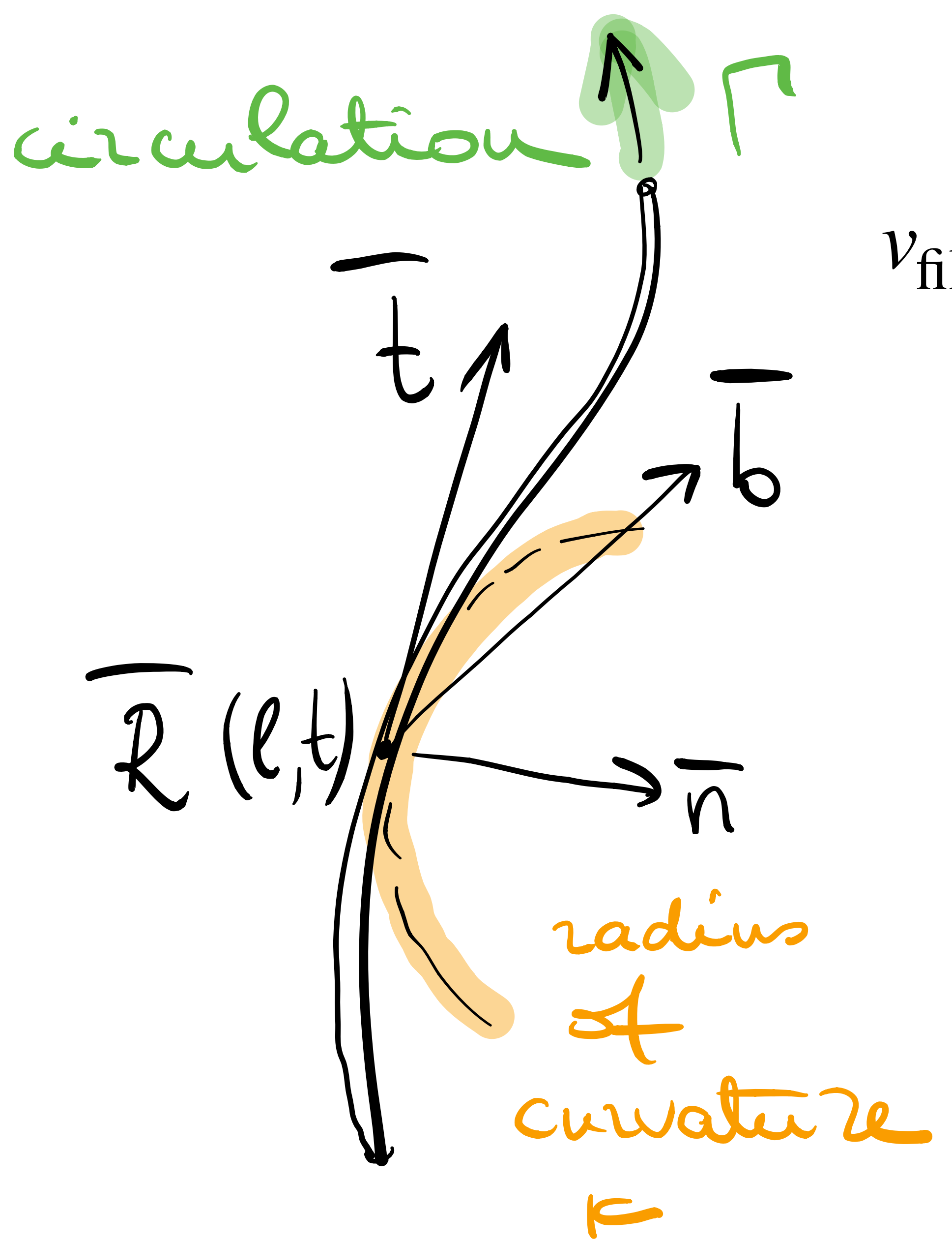
Vortex reconnections in superfluid liquid helium (top) and BEC of cold gases (bottom)



[Serafini et al., PRL 2015]

The Biot-Savart (BS) model is formally derived by the incompressible Euler's equation with filamentary vorticity (in 2D is the point vortex model)

[Saffman, Vortex Dynamics ; Pismen, Vortices in Nonlinear Fields]



$$v_{\mathrm{fil}}(\mathbf{x},t) = -\frac{\Gamma}{4\pi}\int_{\mathscr{L}} \frac{\left[\mathbf{x}-\mathbf{R}(\ell,t)\right]\times d\mathbf{R}(\ell,t)}{\left|\mathbf{x}-\mathbf{R}(\ell,t)\right|^3}$$

Local induction approximation (LIA)

$$\dot{\mathbf{R}}(t) = \frac{\Gamma}{4\pi}\left[\ln\left(\frac{L_0}{a_0}\right) + \mathscr{O}(1)\right]\kappa\hat{\mathbf{b}}$$

MATHEMATICAL MODELS: GROSS-PITAEVSKII (GP)

Derived independently by Gross and Pitaevskii in the 1960s

[Pitaevskii & Stringari, 2003]

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g\,|\psi|^2\psi=0$$

- This is nothing but the nonlinear Scrhoedinger equation (water waves, nonlinear optics, cosmic strings)
- Integrable only in one spatial dimensions
- In more than one spatial dimensions, GP conserves particles (number of bosons), linear momentum and energy, that is

$$N=\int|\psi|^2dV,\quad \mathbf{P}=\frac{i\hbar}{2}\int\left(\psi\nabla\psi^*-\psi^*\nabla\psi\right)dV \quad \text{and}$$

$$H=\int\frac{\hbar^2}{2m}|\nabla\psi|^2+\frac{g}{2}|\psi|^4dV$$

MATHEMATICAL MODELS: GROSS-PITAEVSKII (GP)

Derived independently by Gross and Pitaevskii in the 1960s

[Pitaevskii & Stringari, 2003]

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi - g\,|\psi|^2\psi = 0$$

- Uniform solution $|\psi_0| = \sqrt{\rho_0/m}$
- The healing length $\xi = \sqrt{\hbar^2/(2mg\rho_0)}$ is the only inherent length scale of the system
- Linearising over the uniform state, the large-scale speed of sound is $c = \sqrt{g\rho_0/m^2}$
- The GP equation can be recasted to

$$\boxed{i\frac{\partial\psi}{\partial t} = \frac{c}{\sqrt{2}\xi}\left(-\xi^2\nabla^2\psi + \frac{m}{\rho_0}|\psi|^2\psi\right)}$$

MATHEMATICAL MODELS: GROSS-PITAEVSKII (GP)

$$i\frac{\partial \psi}{\partial t} = \frac{c}{\sqrt{2}\xi}\left(-\xi^2 \nabla^2 \psi + \frac{m}{\rho_0}|\psi|^2\psi\right)$$

Using Madelung transformation $\psi = \sqrt{\rho/m}\exp[i\phi/(\sqrt{2}c\xi)]$ and defining density and velocity as ρ and $\mathbf{v} = \nabla\phi$, respectively, then

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{c^2}{\rho_0}\nabla\rho + c^2\xi^2\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

- The GP models an inviscid, barotropic, and irrotational fluid
- The last term of the second equation, the quantum pressure, becomes negligible at scales larger than the healing length ξ

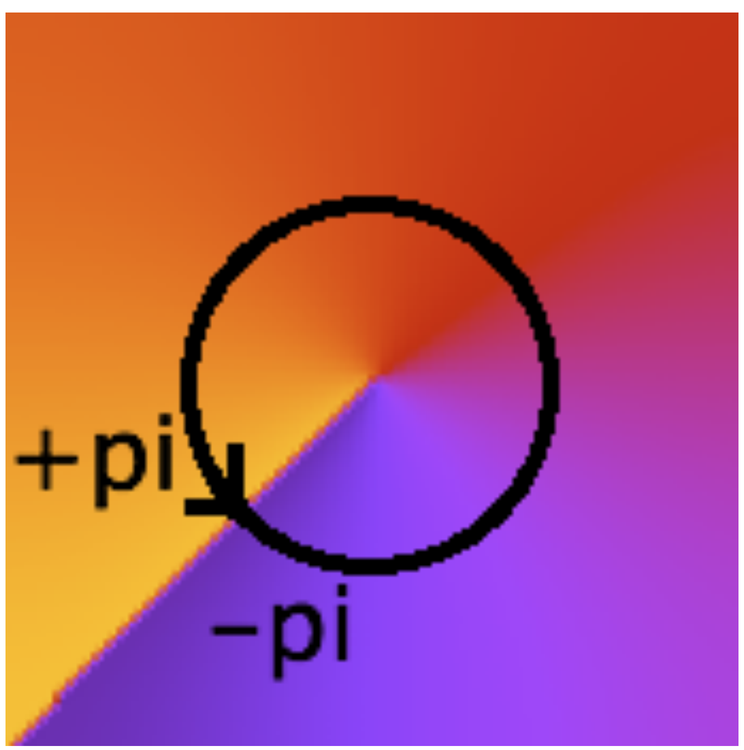
[Nore et al., Phys. Fluids 1997]

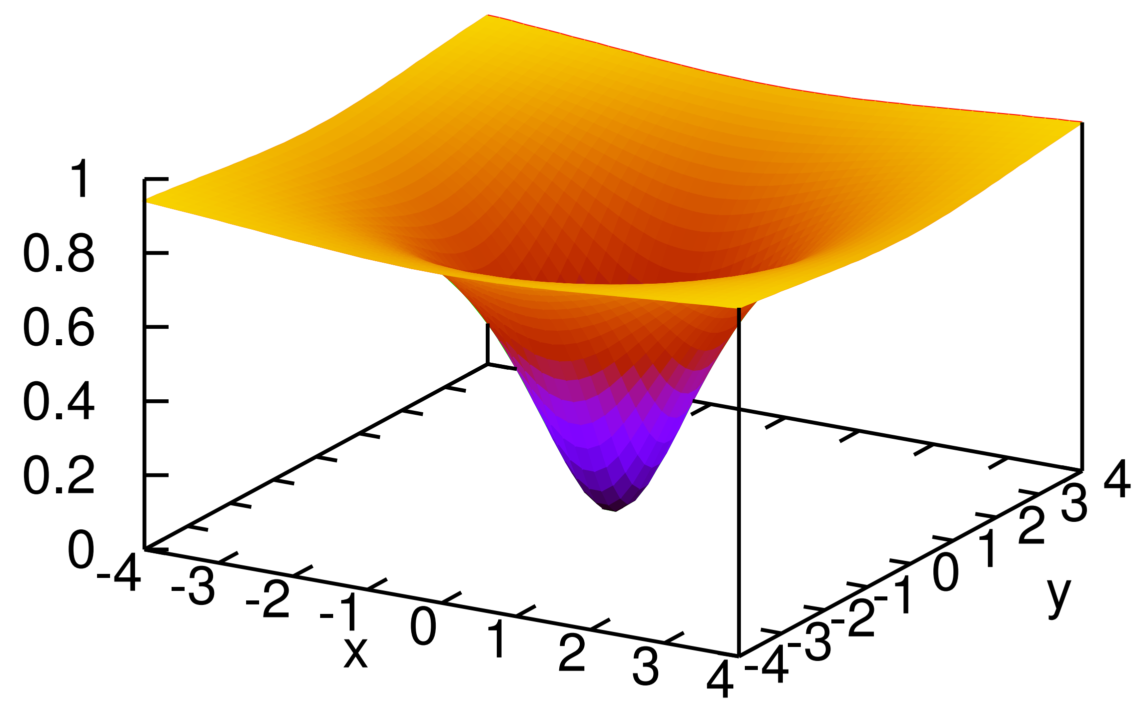
THE GROSS-PITAEVSKII MODEL

$$i\frac{\partial\psi}{\partial t} = \frac{c}{\sqrt{2}\xi}\left(-\xi^2\nabla^2\psi + \frac{m}{\rho_0}|\psi|^2\psi\right)$$

Using Madelung transformation $\psi = \sqrt{\rho/m}\exp[i\phi/(\sqrt{2}c\xi)]$ and defining density and velocity as ρ and $\mathbf{v} = \nabla\phi$, respectively, then

[Pitaevskii, JETP 1961]

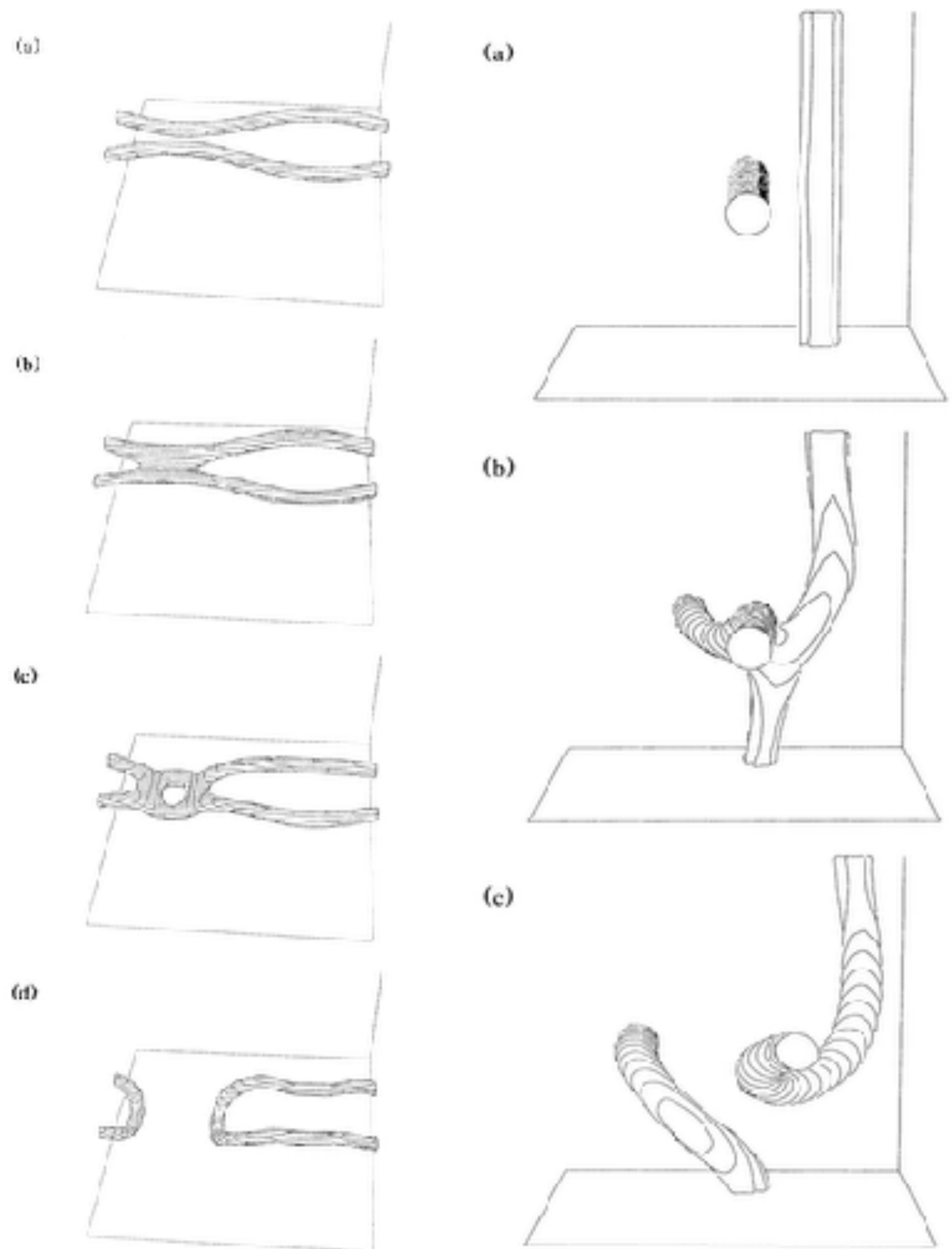




$\arg(\psi)$ profile (in 2d)

$|\psi|^2/\rho_0 = \rho/\rho_0$ profile (in 2d)

VORTEX RECONNECTIONS IN GP

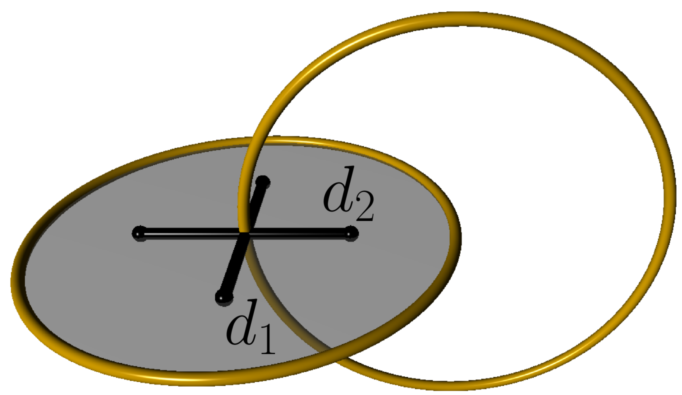


[Koplik & Levine, PRL 1993]

- Vortices naturally reconnect in GP
- Kelvin's theorem does not apply due to density depletion at the vortex core (quantum pressure term)
- Numerically, it is quite easy to prescribe any filamentary initial configuration in the GP model

OUR NUMERICAL EXPERIMENTS IN GP

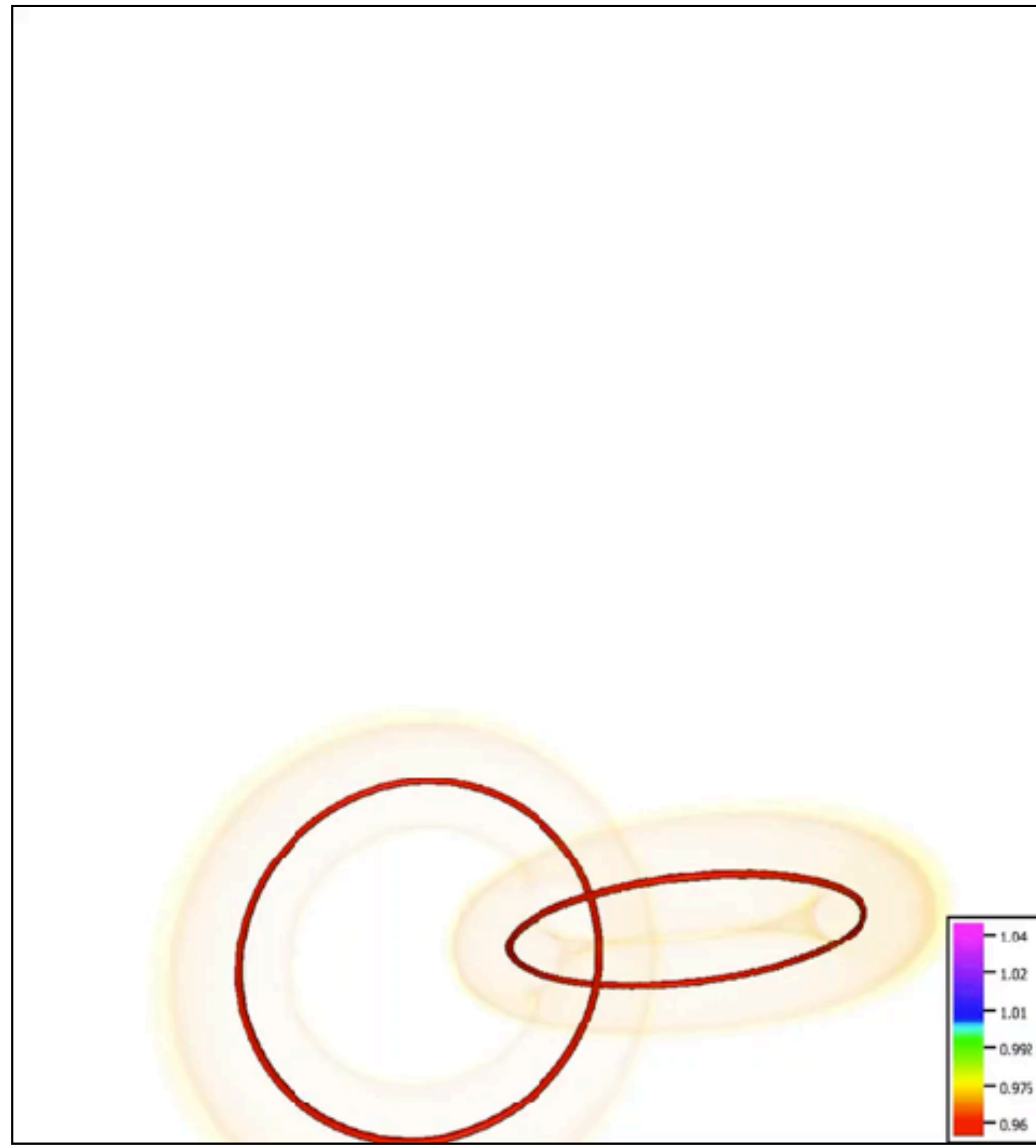
- decay of two linked rings (Hopf link)



- vary the offset parameters (d_1, d_2), spanning over 49 different configurations
- track accurately the positions of the vortex filaments

[Villois et al., JPhysA 2016]

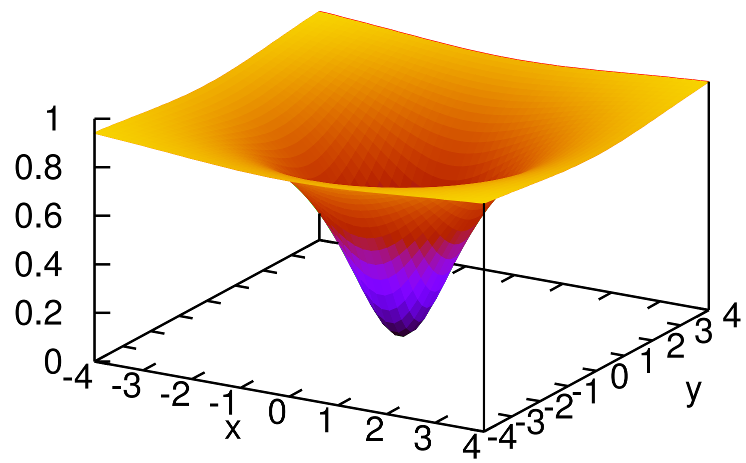
Example of the evolution of the density field ρ of an Hopf link realisation



ABOUT RECONNECTION: LINEAR THEORY APPROXIMATION

[Nazarenko & West, JLTP 2003]

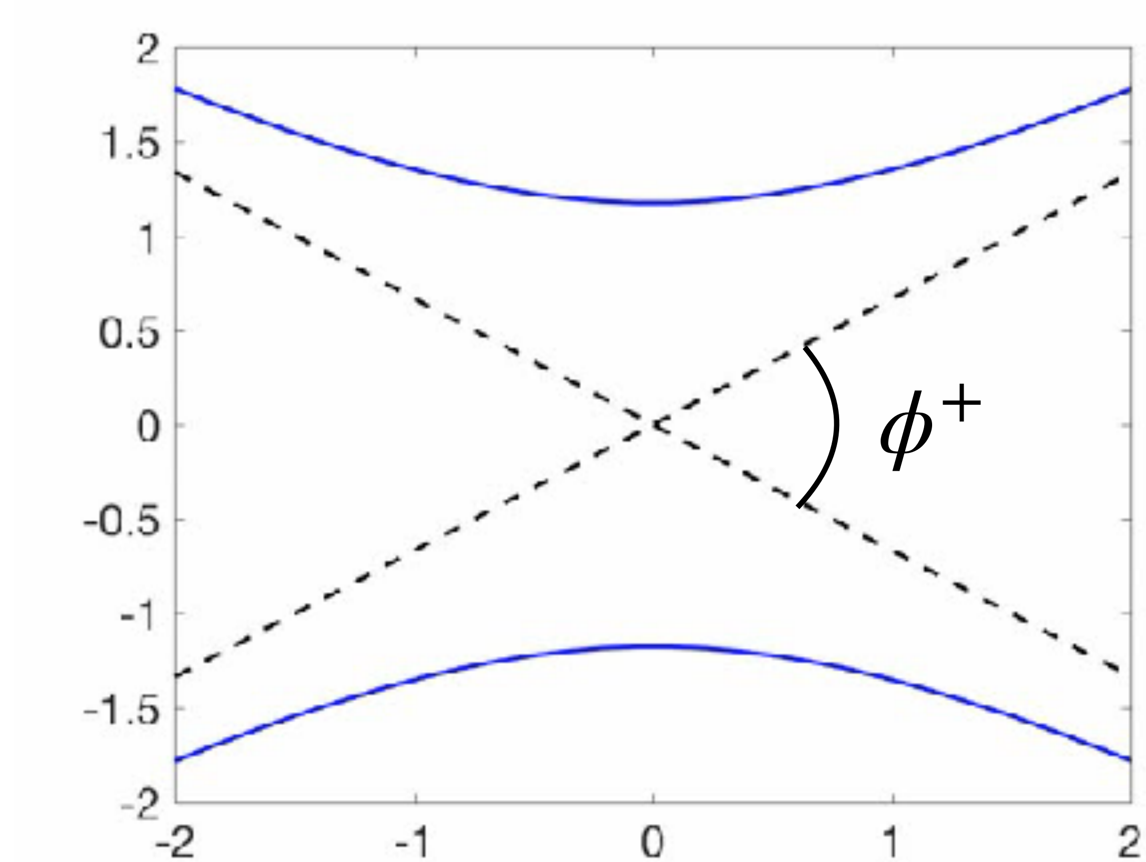
$$\delta^{\pm}(t) \le \xi \quad \Longrightarrow \quad i\frac{\partial \psi}{\partial t} = \frac{c}{\sqrt{2}\xi}\left(-\xi^2 \nabla^2 \psi + \cancel{\frac{m}{\rho_0}|\psi|^2 \psi}\right)$$



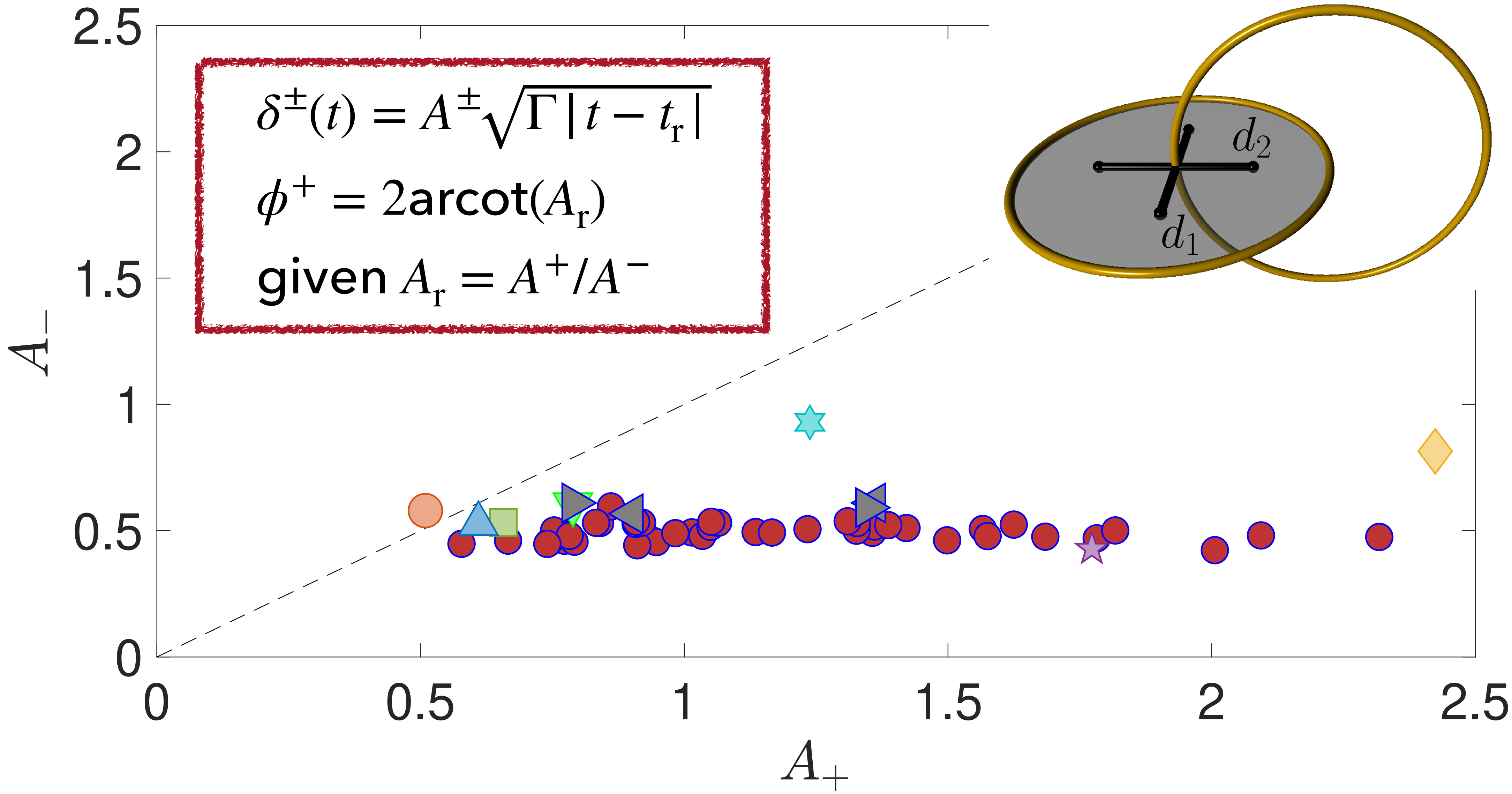
$$\boxed{\begin{array}{l} \delta^{\pm}(t) = A^{\pm}\sqrt{\Gamma\,|\,t - t_{\mathrm{r}}\,|} \\ \phi^{+} = 2\mathrm{arcot}(A_{\mathrm{r}})\,, \quad \text{where} \quad A_{\mathrm{r}} = A^{+}/A^{-} \end{array}}$$

[Villois et al., PRFluids 2017]

- same scaling $\delta \propto t^{1/2}$ before and after, only the pre-factors change $A^{\pm}$
- filaments reconnect tangent to a plane and their projections are branches of an hyperbola



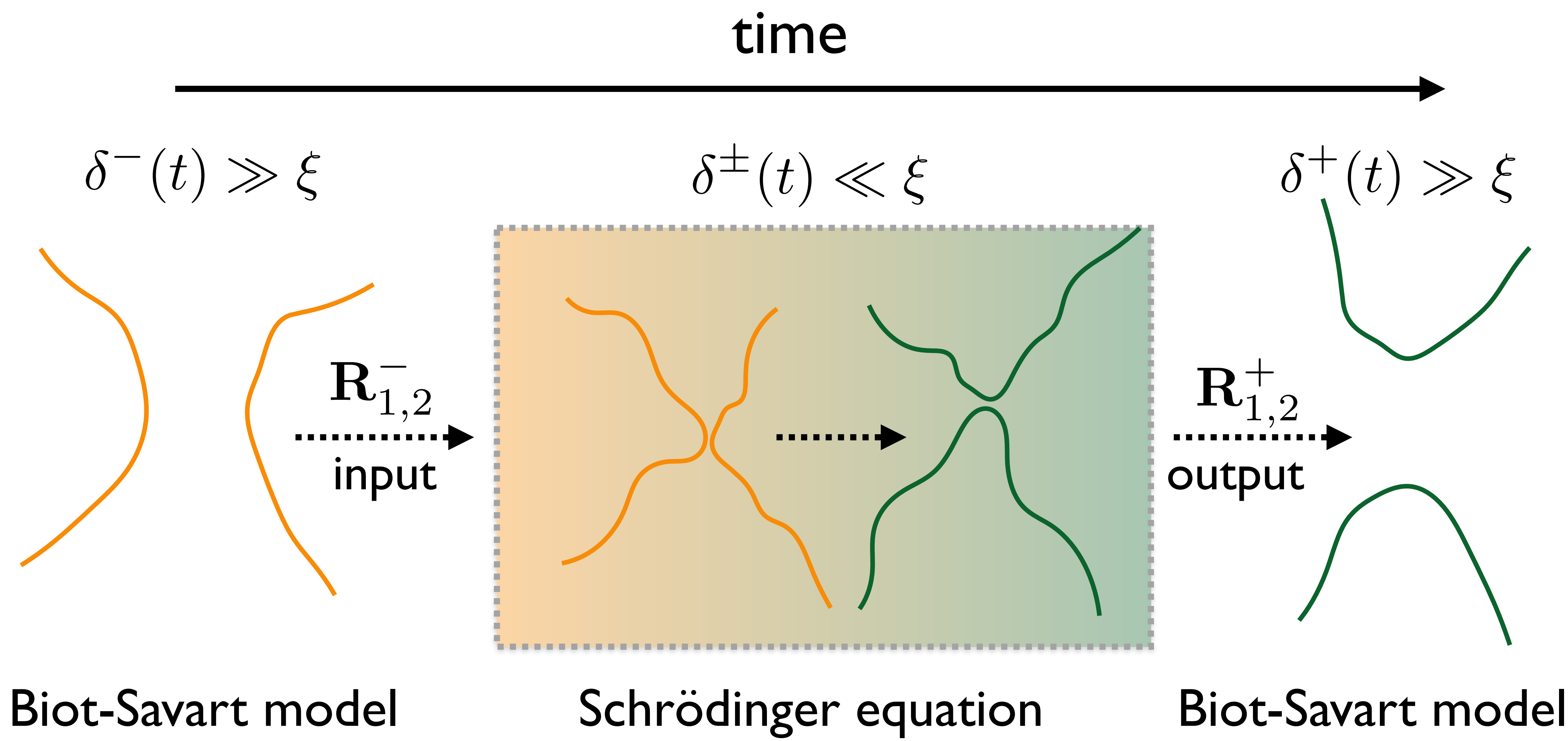
ASYMMETRY IN THE DISTRIBUTION OF THE RATES $A^{\pm}$



Red points data of this work, other symbols are from [Villois et al., PRFluids 2018] and [Galantucci et al., PNAS 2019]

CLEAR EVIDENCE OF IRREVERSIBLE DYNAMICS, EVEN IF THE GP MODEL IS TIME-REVERSIBLE. HOW TO EXPLAIN THIS ASYMMETRY?

OUR MATCHING THEORY



- when $\delta(t) \leq \delta_{\text{lin}}$ linear theory (linear Schrödinger)
- when $\delta(t) \geq \delta_{\text{lin}}$ nonlinear theory using vortex filament model or local induction approximation (LIA)
- matching of the two theories at $\delta(t) = \delta_{\text{lin}}$

ABOUT THE RECONNECTION: THE LINEAR THEORY

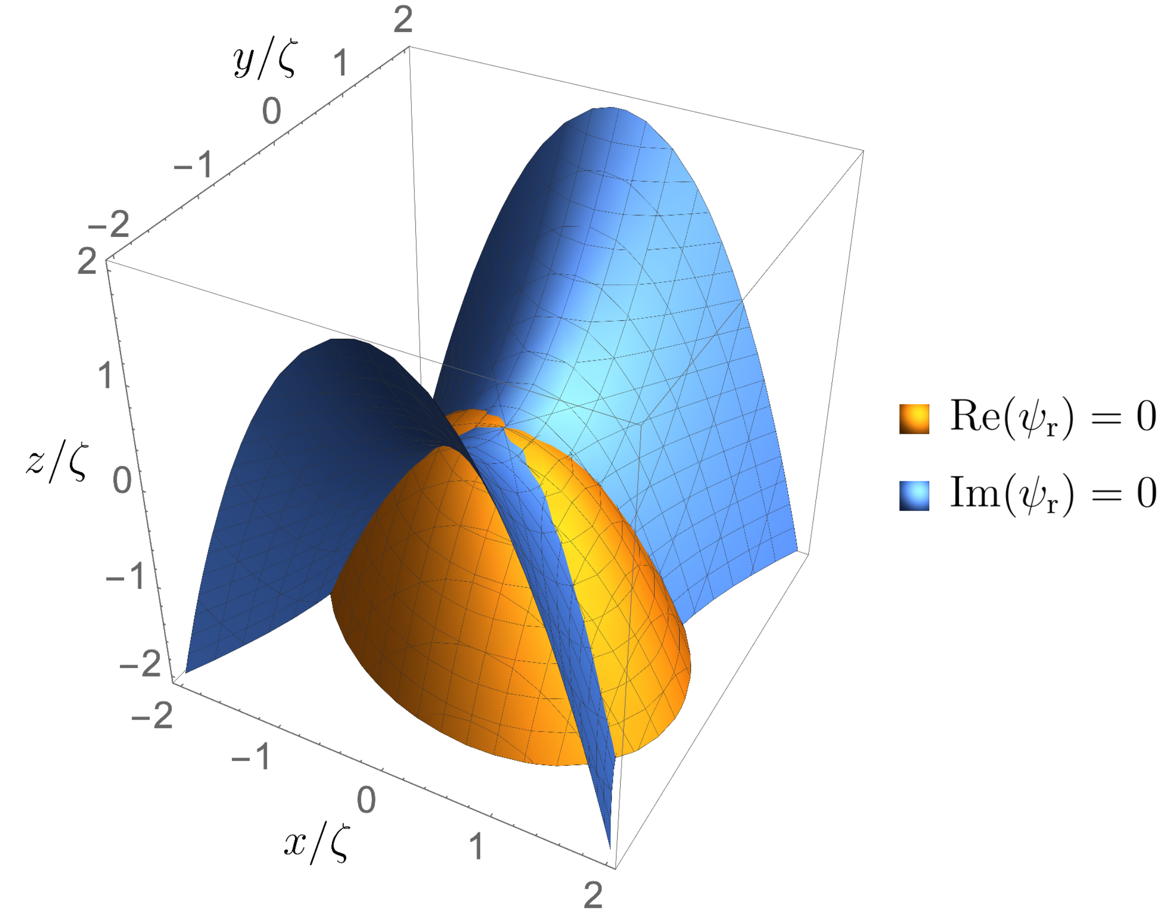
$$i\frac{\partial\psi}{\partial t} = -\frac{\Gamma}{4\pi}\nabla^2\psi$$

A general second-order polynomial solution at the reconnection time $t_\mathrm{r} = 0$ having two nodal-lines (vortices) is given by

$$\psi_\mathrm{r}(x,y,z) = \frac{1}{\zeta^{5/2}}\left\{p\left[z - \frac{A(x\cos\theta + y\sin\theta)^2 + B(-x\cos\theta + y\sin\theta)^2}{2\zeta}\right] + i\left[z - \frac{Cx^2 + Dy^2}{2\zeta}\right]\right\}$$

$p = \pm 1$, $(A, B, C, D) \in \mathbb{R}$,
$\theta \in [0,\pi]$, $\zeta > 0$ is a generic length scale

- The vortices are identified as the intersection of $Re(\psi_\mathrm{r}) = 0$ and $Im(\psi_\mathrm{r}) = 0$
- Without any loss of generality, we set $\theta = 0$ as this is a quadratic form

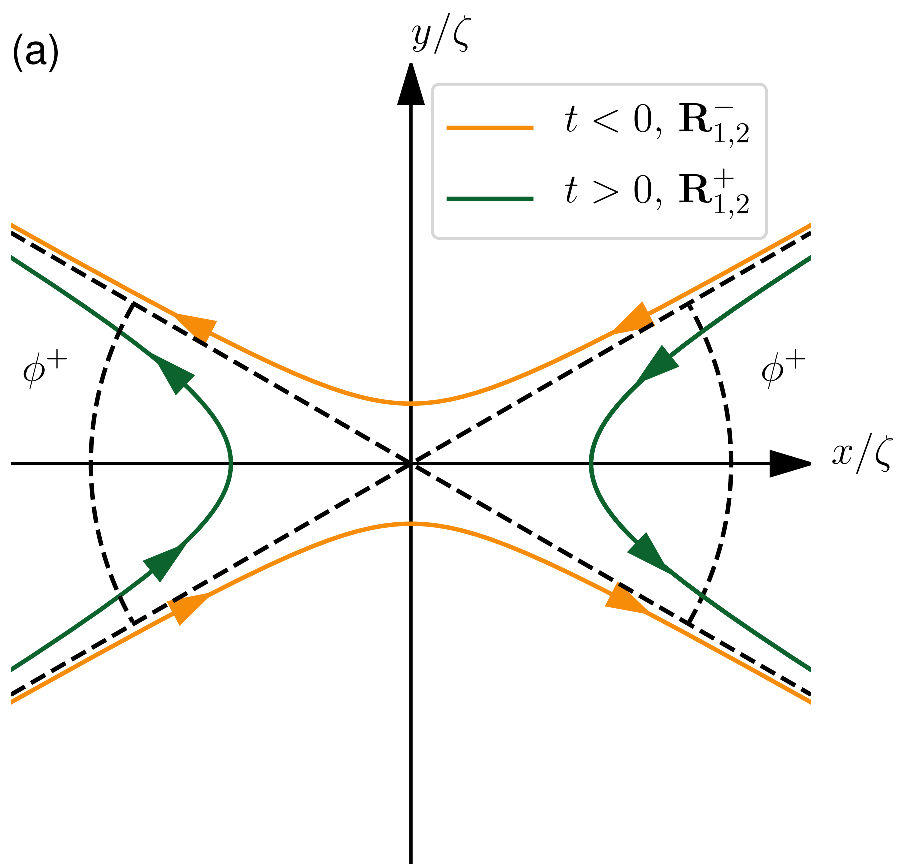


ABOUT THE RECONNECTION: THE LINEAR THEORY

Once evolved in time, the wave-function reads

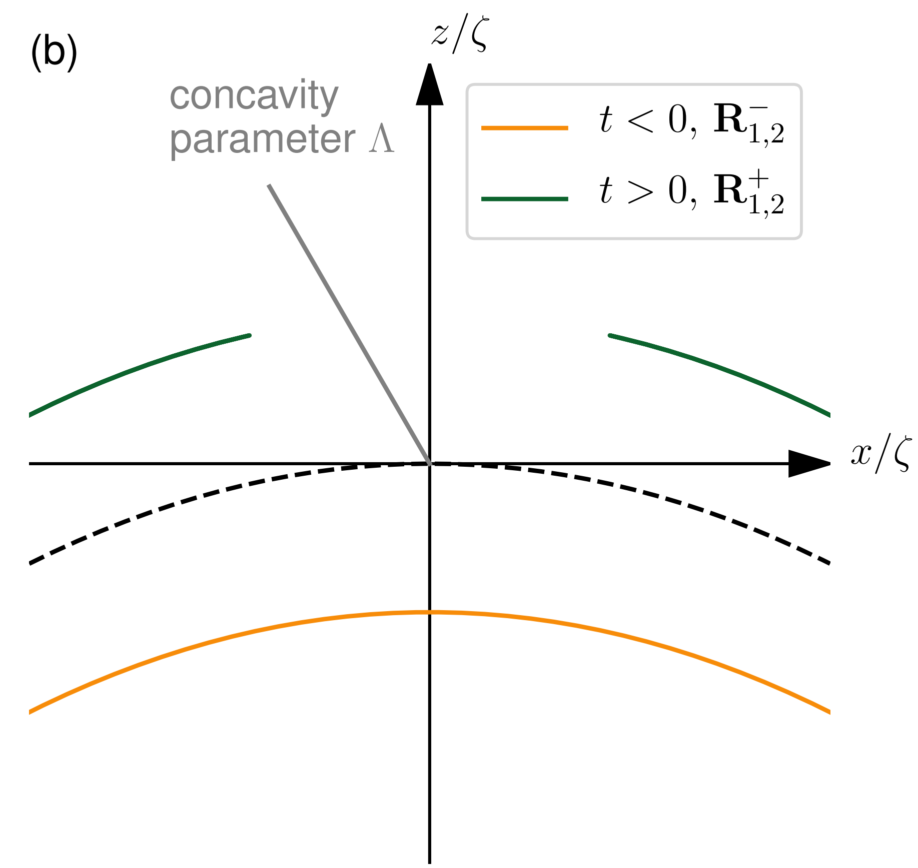
$$i\frac{\partial \psi}{\partial t} = -\frac{\Gamma}{4\pi}\nabla^2\psi \quad \Longrightarrow \quad \psi(x,y,z,t) = \left(1 + it\frac{\Gamma}{4\pi}\nabla^2\right)\psi_{\mathrm{r}}(x,y,z)$$

projections onto the $z=0$ plane



$$\boxed{-\frac{C-A}{B-D}x^2 + y^2 = \frac{A+B+C+D}{2(B-D)p\pi}\Gamma t}$$

projections onto the $y=0$ plane

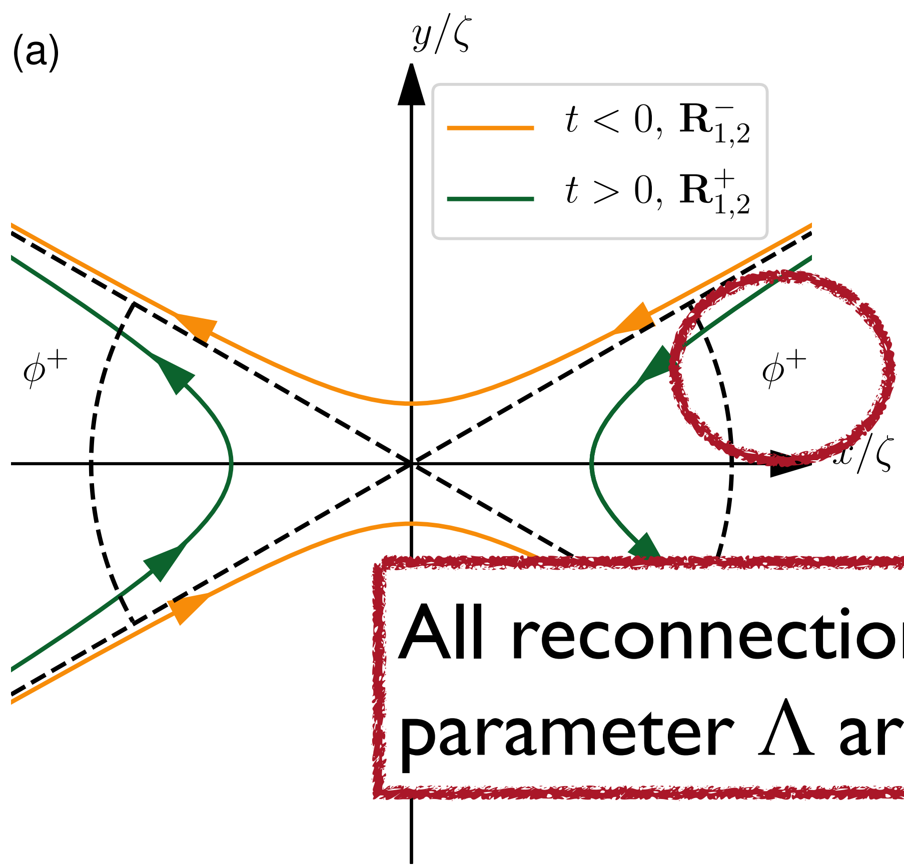


$$\boxed{z = \frac{BC-AD}{2(B-D)\zeta}x^2 + \frac{D(C+D)+B(A+B)}{4(B-D)p\pi\zeta}\Gamma t}$$

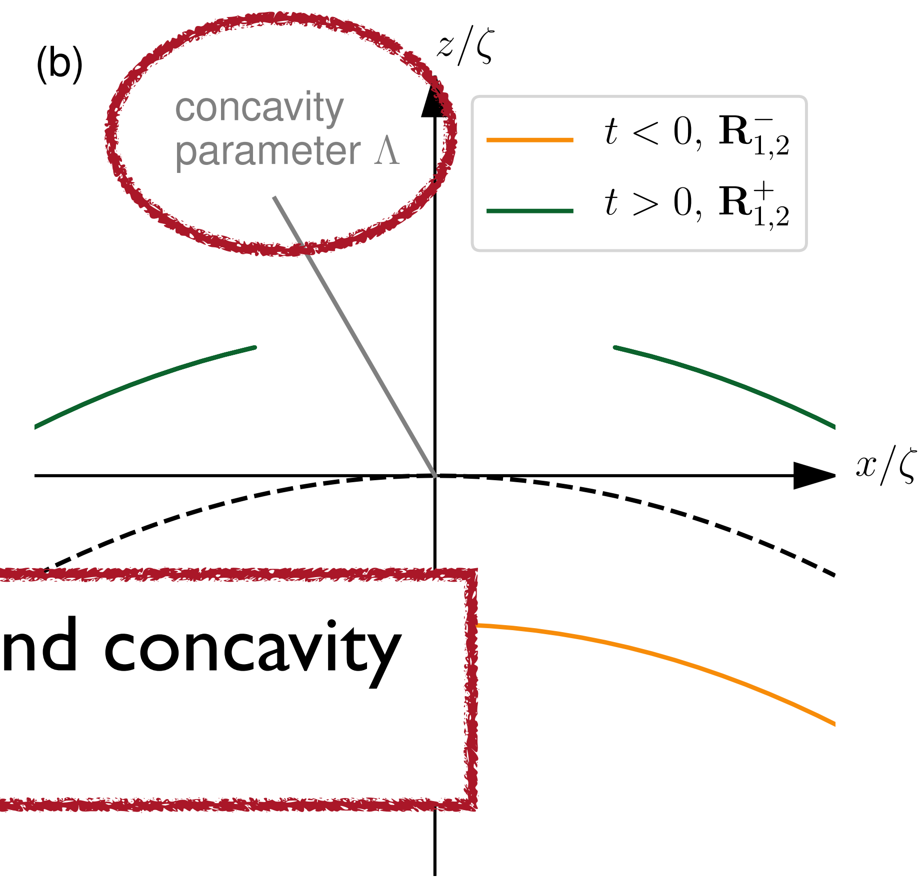
ABOUT THE RECONNECTION: THE LINEAR THEORY

$$\left\{2\Lambda < \left[\tan^2\left(\frac{\phi^+}{2}\right) - 1\right](B+D)\right\} \cap \left[\left(p = -1 \cap D > B\right) \cup \left(p = 1 \cap D < B\right)\right]$$

projections onto the $z = 0$ plane



projections onto the $y = 0$ plane

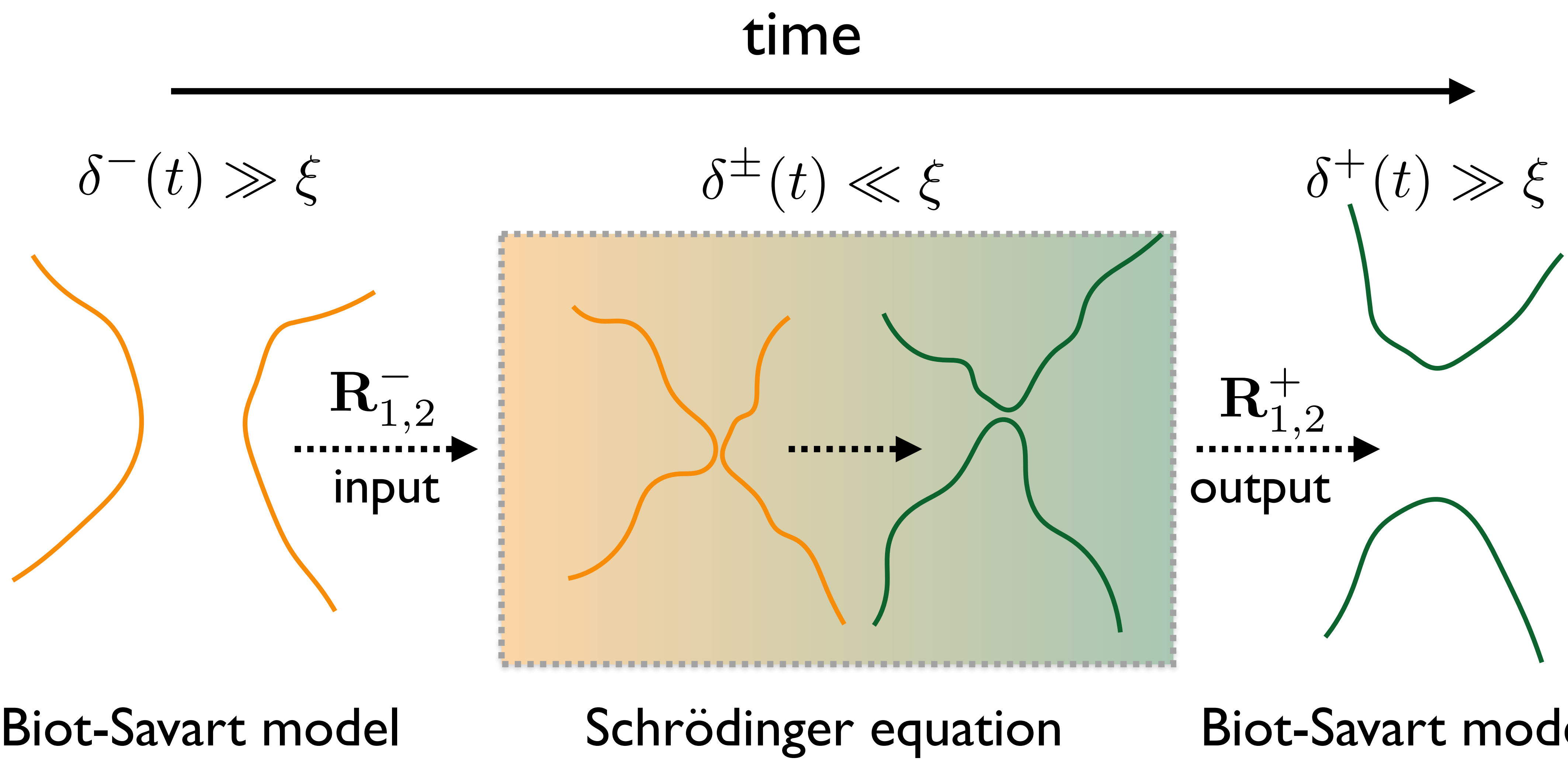


All reconnection angles ϕ^+ and concavity parameter Λ are possible!

$$-\frac{C-A}{B-D}x^2 + y^2 = \frac{A+B+C+D}{2(B-D)p\pi}\Gamma t$$

$$z = \frac{BC-AD}{2(B-D)\zeta}x^2 + \frac{D(C+D)+B(A+B)}{4(B-D)p\pi\zeta}\Gamma t$$

OUR MATCHING THEORY



- matching of the two theories at $\delta(t) = \delta_{\mathrm{lin}}$
- in BS (and LIA) theory

momentum: $\mathbf{P}^\pm_{\mathrm{fil}} = \dfrac{\kappa}{2} \displaystyle\int_{\mathscr{L}} \mathbf{R}^\pm \times d\mathbf{R}^\pm$

energy: $E^\pm_{\mathrm{LIA}} \propto \displaystyle\int_{\mathscr{L}} |\, d\mathbf{R}^\pm \,|$

$\Longrightarrow$

$$\boxed{\begin{aligned} \Delta \mathbf{P}_{\mathrm{fil}} &= \mathbf{P}^+_{\mathrm{fil}} - \mathbf{P}^-_{\mathrm{fil}} \\ \Delta E_{\mathrm{LIA}} &= E^+_{\mathrm{LIA}} - E^-_{\mathrm{LIA}} \end{aligned}}$$

[Pismen, 1999]

OUR MATCHING THEORY

- A useful parametrisation for the filaments, in terms of ϕ^+ and Λ, so that they satisfy the shape found in the linear theory is

$$
\begin{aligned}
\mathbf{R}_1^-(\ell,t) &= \left\{ -\frac{\delta^-(t)}{2}\cot\left(\frac{\phi^+}{2}\right)\sinh(\ell), \frac{\delta^-(t)}{2}\cosh(\ell), z^-(\ell,t) \right\} \\
\mathbf{R}_2^-(\ell,t) &= = \left\{ \frac{\delta^-(t)}{2}\cot\left(\frac{\phi^+}{2}\right)\sinh(\ell), -\frac{\delta^-(t)}{2}\cosh(\ell), z^-(\ell,t) \right\} \\
\mathbf{R}_1^+(\ell,t) &= \left\{ -\frac{\delta^+(t)}{2}\cosh(\ell), \frac{\delta^+(t)}{2}\tan\left(\frac{\phi^+}{2}\right)\sinh(\ell), z^+(\ell,t) \right\} \\
\mathbf{R}_2^+(\ell,t) &= \left\{ \frac{\delta^+(t)}{2}\cosh(\ell), -\frac{\delta^+(t)}{2}\tan\left(\frac{\phi^+}{2}\right)\sinh(\ell), z^+(\ell,t) \right\}
\end{aligned}
\quad , \quad \text{where } l \in \mathbb{R}
$$

- matching of the two theories at $\delta(t) = \delta_{\text{lin}}$
- in BS (and LIA) theory

momentum: $\mathbf{P}^{\pm}_{\text{fil}} = \frac{\kappa}{2}\int_{\mathscr{L}} \mathbf{R}^{\pm} \times d\mathbf{R}^{\pm}$

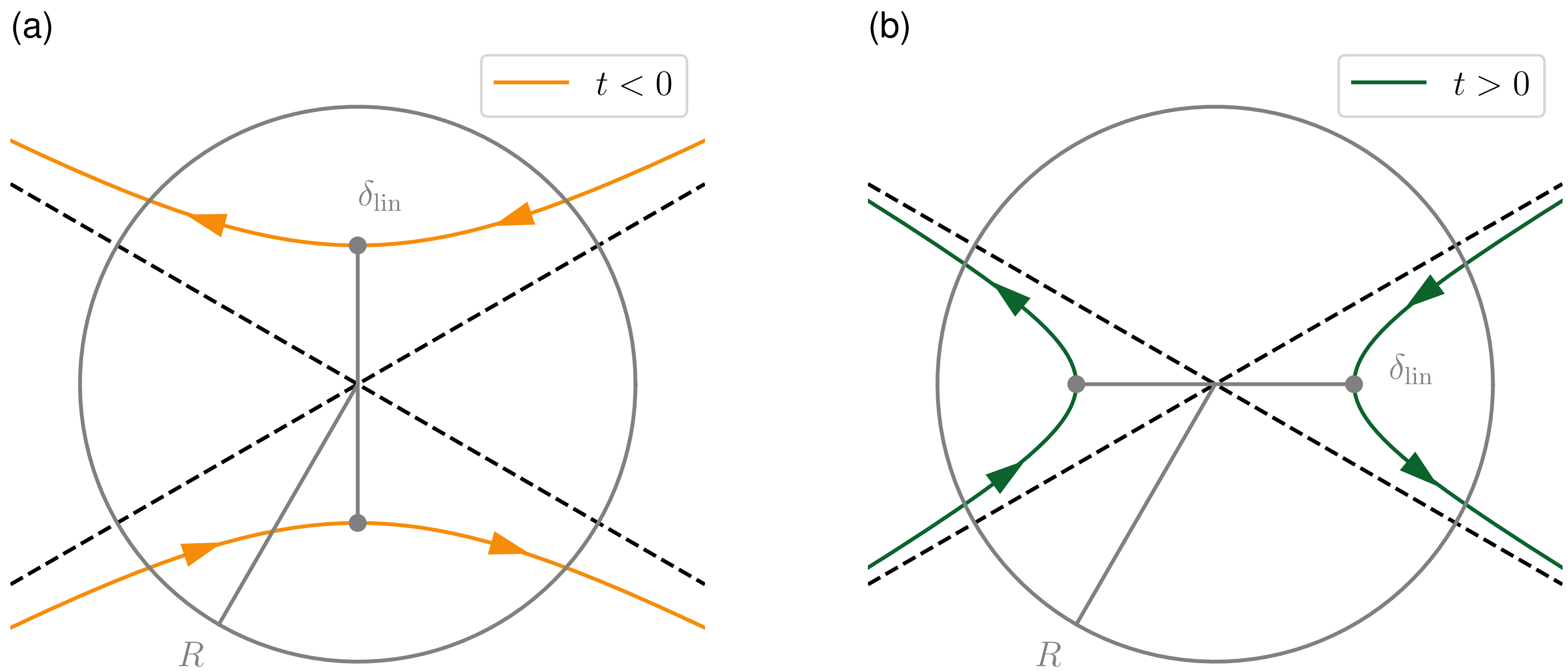
energy: $E^{\pm}_{\text{LIA}} \propto \int_{\mathscr{L}} |d\mathbf{R}^{\pm}|$

$$\Longrightarrow \boxed{\begin{aligned} \Delta\mathbf{P}_{\text{fil}} &= \mathbf{P}^{+}_{\text{fil}} - \mathbf{P}^{-}_{\text{fil}} \\ \Delta E_{\text{LIA}} &= E^{+}_{\text{LIA}} - E^{-}_{\text{LIA}} \end{aligned}}$$

[Pismen, 1999]

THE BS (AND LIA) REGIME

As the filaments are branches of hyperbola they are of infinite length. We compute their integrals in a finite cylinder parallel to the z-axis, centred at the reconnection point (the origin) and of radius $R > \delta_{\text{lin}}$



The limits of integration, in the parametrisation of the filaments, are given by

$$L^-(R/\delta_{\text{lin}}) = \frac{1}{2}\ln\left\{\frac{8(R/\delta_{\text{lin}})^2 + (A_r^2 - 1) + 2\sqrt{\left[4\left(R/\delta_{\text{lin}}\right)^2 - 1\right]\left[4\left(R/\delta_{\text{lin}}\right)^2 + A_r^2\right]}}{A_r^2 + 1}\right\}$$

$$L^+(R/\delta_{\text{lin}}) = \frac{1}{2}\ln\left\{\frac{8A_r^2(R/\delta_{\text{lin}})^2 + (1 - A_r^2) + 2A_r\sqrt{\left[4\left(R/\delta_{\text{lin}}\right)^2 - 1\right]\left[4A_r^2\left(R/\delta_{\text{lin}}\right)^2 + 1\right]}}{A_r^2 + 1}\right\}$$

THE BS (AND LIA) REGIME

momentum: $\mathbf{P}^{\pm}_{\rm fil} = \dfrac{\kappa}{2}\displaystyle\int_{\mathscr{L}} \mathbf{R}^{\pm} \times d\mathbf{R}^{\pm}$

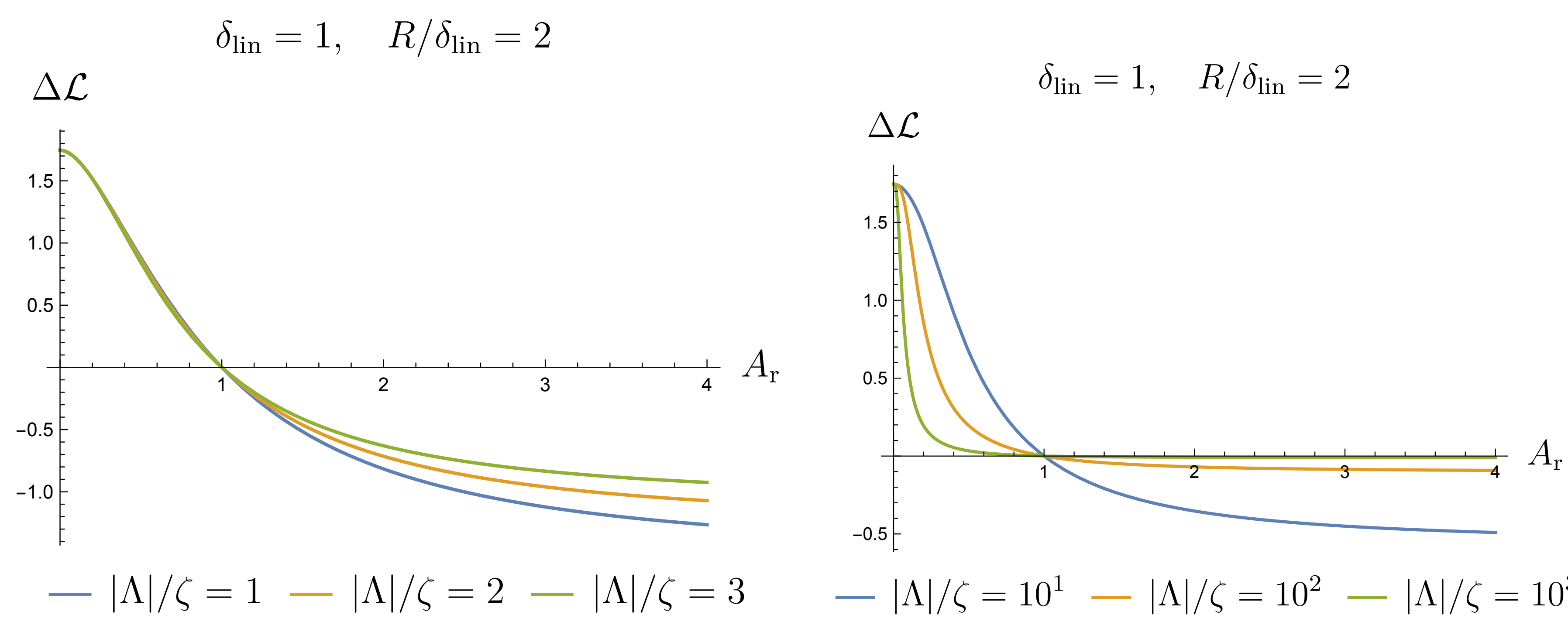
energy: $E^{\pm}_{\rm LIA} \propto \displaystyle\int_{\mathscr{L}} |d\mathbf{R}^{\pm}|$

$$\Longrightarrow \boxed{\begin{aligned}\Delta \mathbf{P}_{\rm fil} &= \mathbf{P}^{+}_{\rm fil} - \mathbf{P}^{-}_{\rm fil} \\ \Delta E_{\rm LIA} &= E^{+}_{\rm LIA} - E^{-}_{\rm LIA}\end{aligned}}$$

$$\Delta \mathbf{P}_{\rm fil} \propto \left(0,0,\frac{1+A_{\rm r}^2}{A_{\rm r}}\right) = \left(0,0,-2\csc\phi^{+}\right)$$

$$\Delta E_{\rm LIA}(A_{\rm r}, \Lambda/\zeta, \delta_{\rm lin}, R/\delta_{\rm lin}) \propto \int_{-L^{+}(R/\delta_{\rm lin})}^{L^{+}(R/\delta_{\rm lin})} \left|\frac{\partial \mathbf{R}^{+}_1}{\partial \ell}\right| + \left|\frac{\partial \mathbf{R}^{+}_2}{\partial \ell}\right| d\ell - \int_{-L^{-}(R/\delta_{\rm lin})}^{L^{-}(R/\delta_{\rm lin})} \left|\frac{\partial \mathbf{R}^{-}_1}{\partial \ell}\right| + \left|\frac{\partial \mathbf{R}^{-}_2}{\partial \ell}\right| d\ell$$

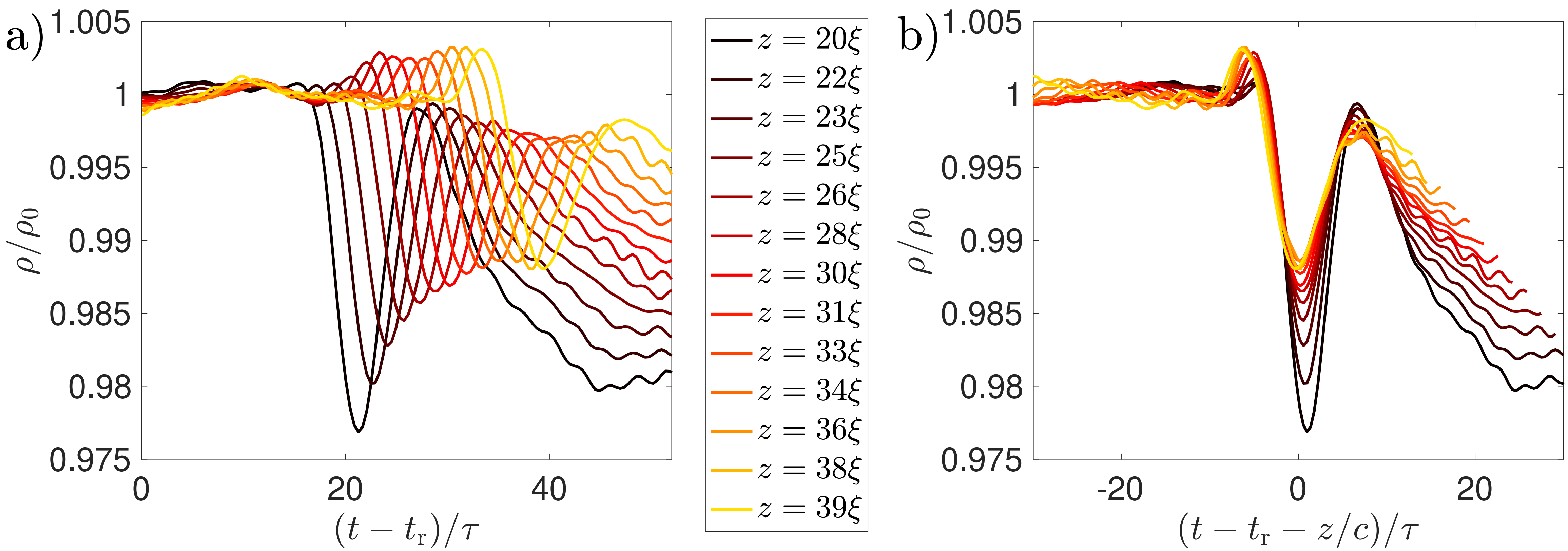
- Computed analytically only for $\Lambda = 0$
- Invariant for $\Lambda \leftrightarrow -\Lambda$
- Converge for large $R/\delta_{\rm lin}$
- Tending to 0 for $|\Lambda| \to \infty$



CONVERSION OF FILAMENT'S MOMENTUM INTO SOUND

$$\mathbf{P}_{\text{pulse}} = -\Delta\mathbf{P}_{\text{fil}} \propto \left(0,\ 0,\ \frac{1+A_{\mathrm{r}}^2}{A_{\mathrm{r}}}\right) \implies \Delta P_{\text{wav,z}} > 0$$

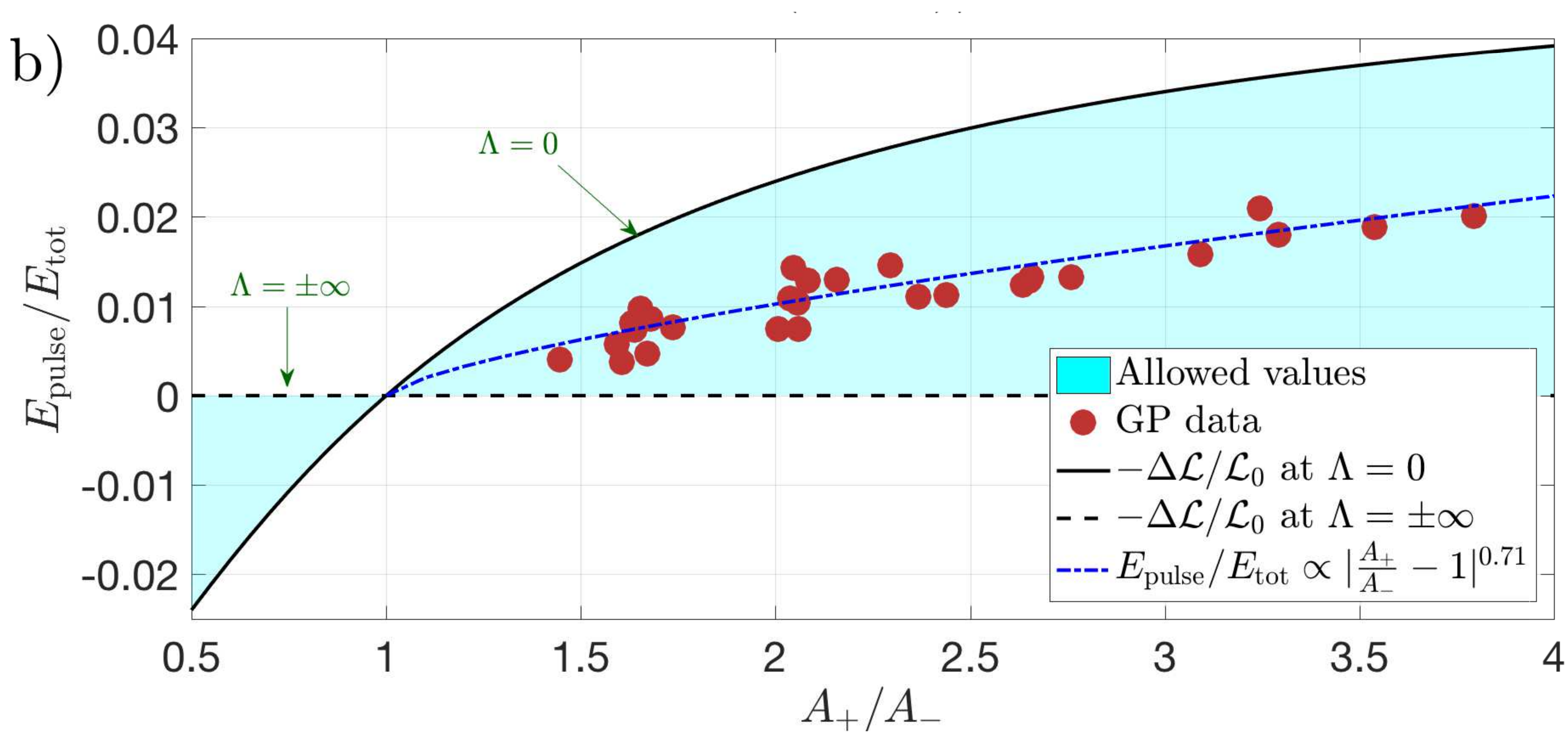
Example of sound pulse emission propagating along the positive z-axis



- propagation at almost speed of sound c
- some dispersive effects

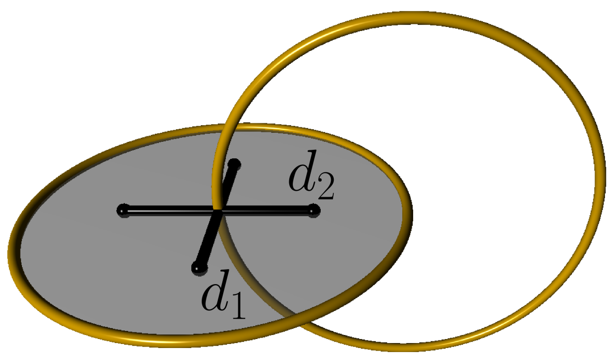
CONVERSION OF FILAMENT'S ENERGY INTO SOUND

$E_{\text{pulse}} = -\Delta E_{\text{LIA}} = \Delta \mathscr{L} / \mathscr{L}_0$, $\mathscr{L}_0$ is the initial length

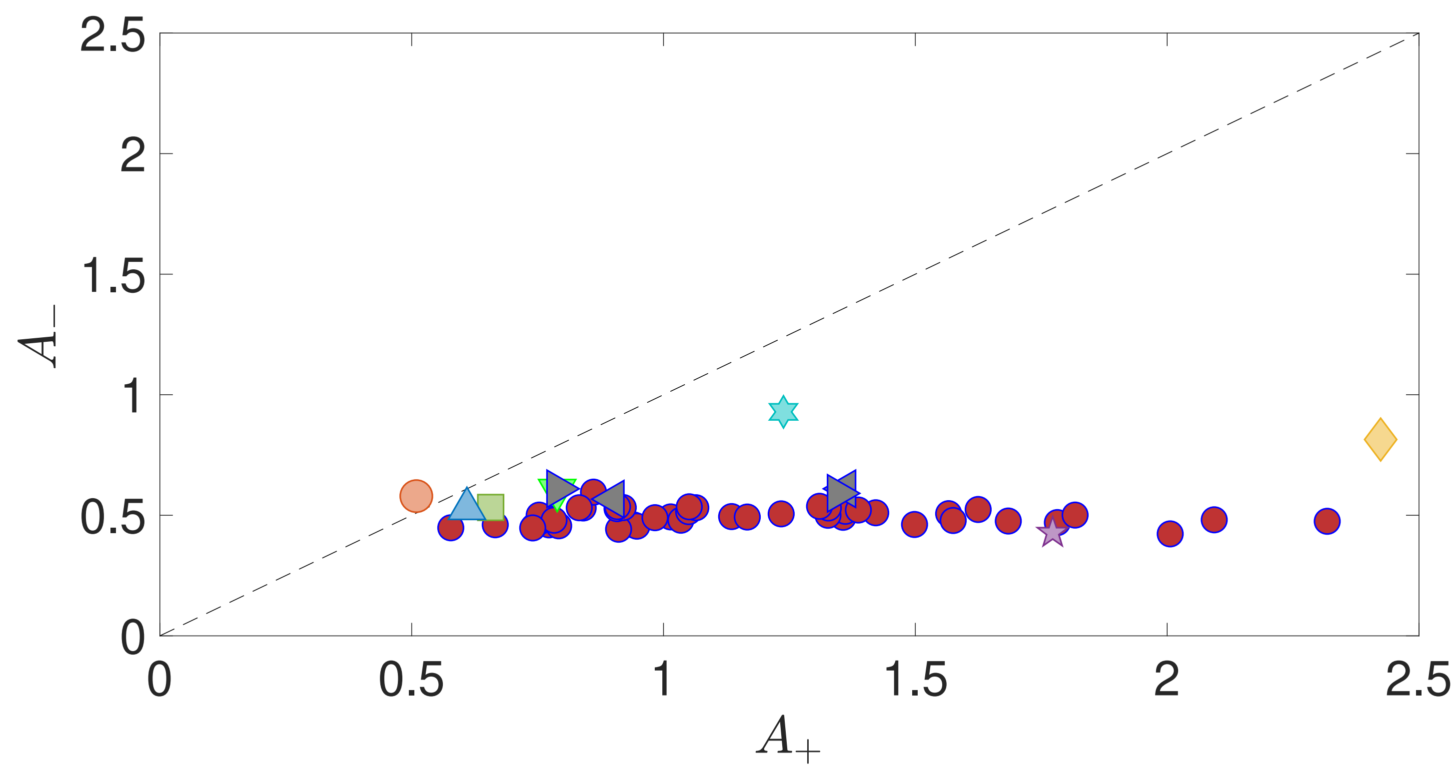


THIS EXPLAIN THE ASYMMETRY IN THE DISTRIBUTION OF $A^{\pm}$ AS SOUND PULSES WITH NEGATIVE ENERGY ARE PHYSICALLY IMPOSSIBLE!

SUMMARY AND CONCLUSIONS

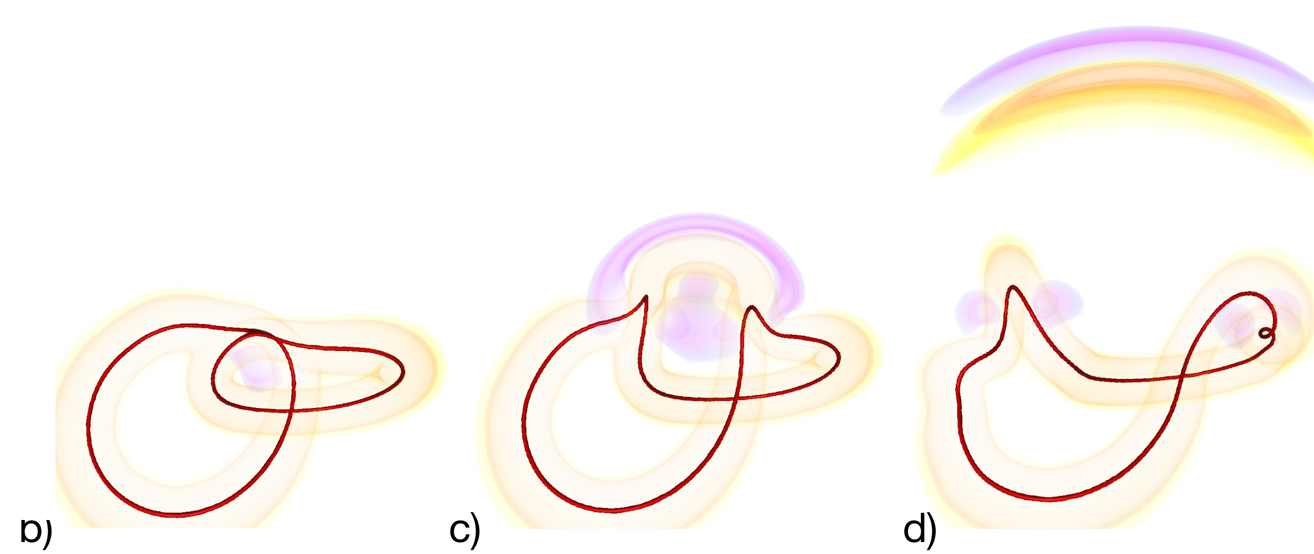


- We performed a statistical study of vortex reconnections in quantum fluids (GP model)

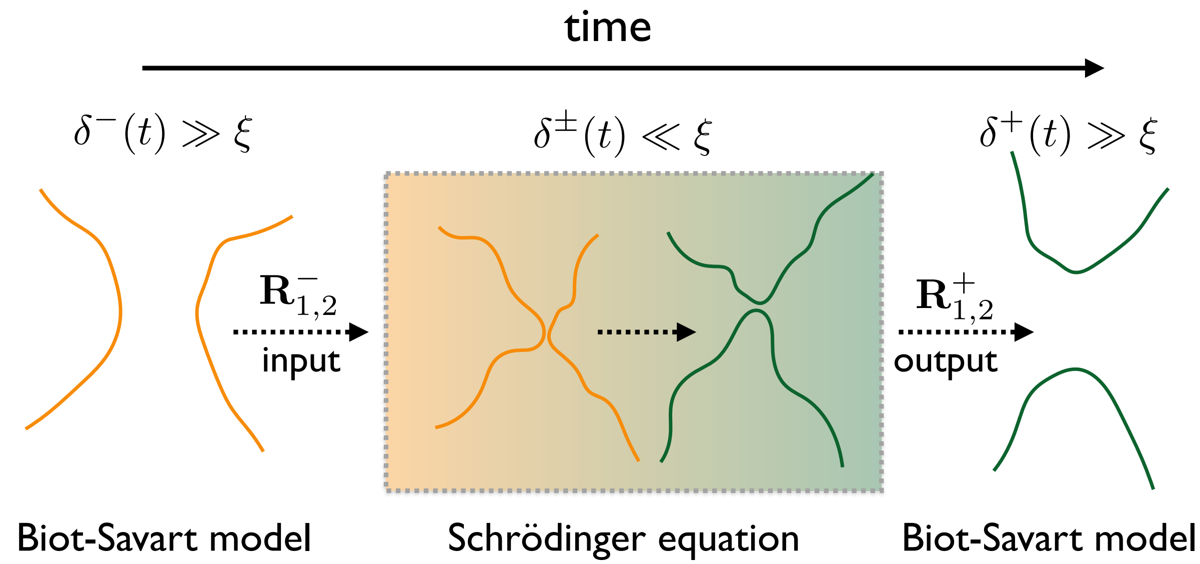


- We found that the distribution of the rates of approach A^- and separation A^+ is asymmetric, evidence of irreversible dynamics

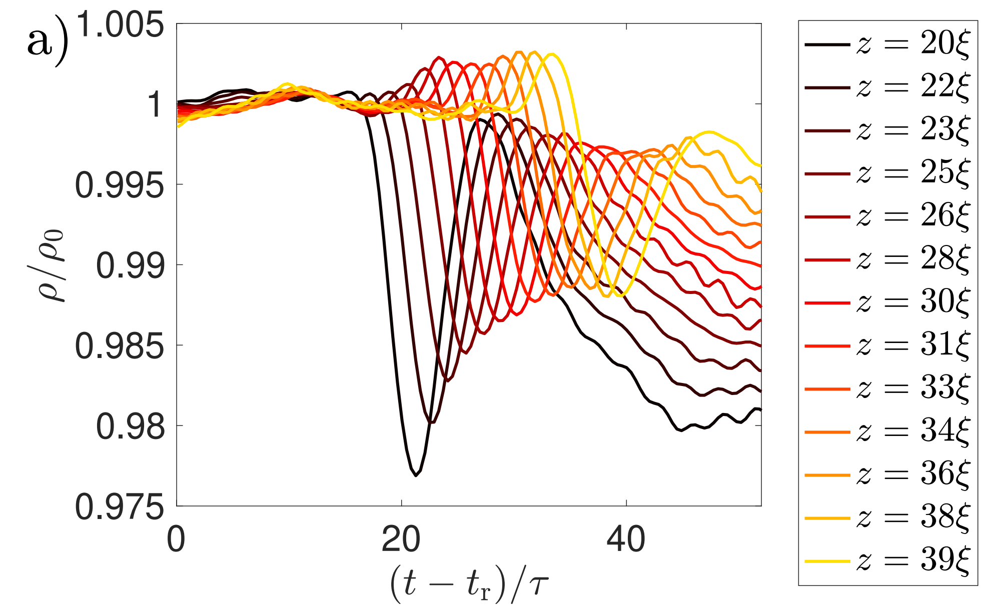
- This is the manifestation of an irreversible dynamics explained by the emission of a sound pulse



SUMMARY AND CONCLUSIONS

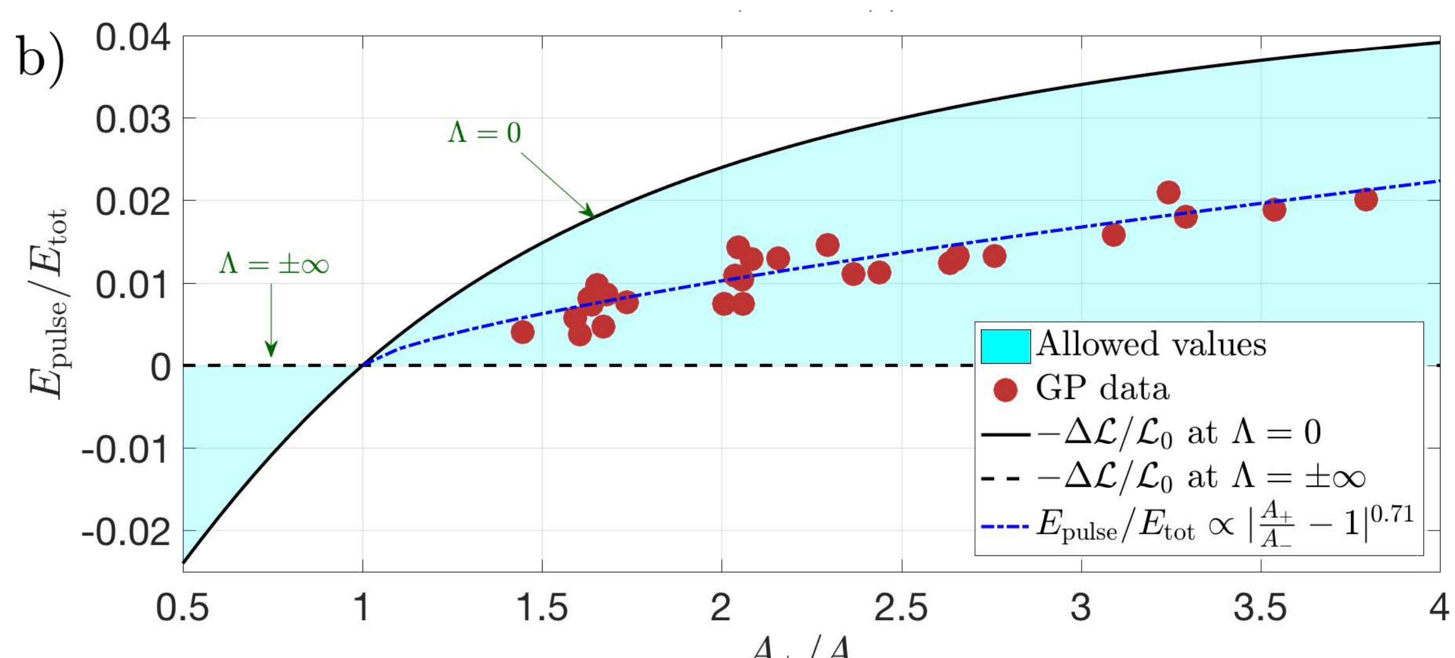


- We proposed a matching between linear theory and BS (and LIA)

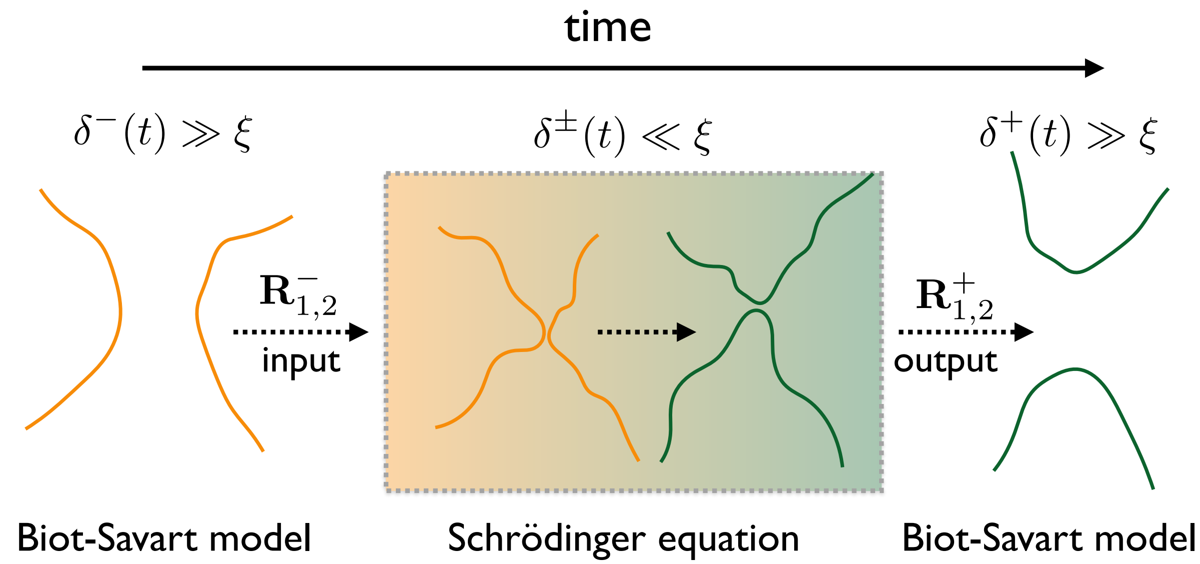


- We found that the momentum of the sound pulse only propagates towards the positive z-axis

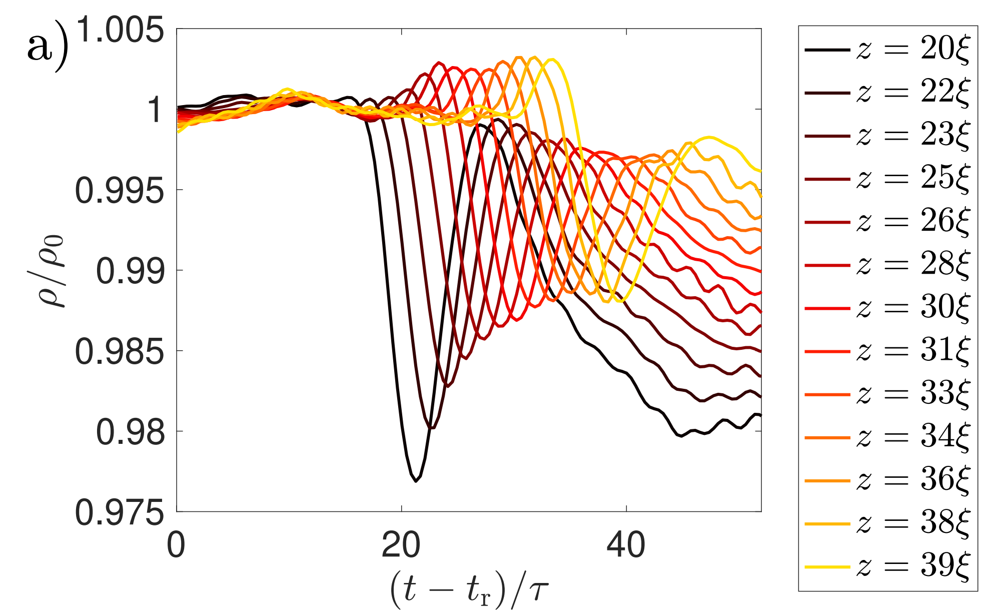
- We quantitative explained the origin of the irreversible dynamics by showing that the energy of the sound pulse is only positive when $A^+ > A^-$ that is for $0 \leq \phi^+ \leq \pi/2$



FUTURE WORKS



- Work on a "more precise" asymptotic matching theory



- Analyse the sound pulse, to know if it is a "superposition" of (quasi-)linear waves, or a full nonlinear structure

- Look at the problem of reconnections in the Euler limit (regularity applied maths problem) by letting different regularisation scales (viscosity in classical fluid, dispersion in quantum fluids) tends to zero

- Assume thermal or turbulent fluctuations to find how the distribution of the rates $A^\pm$ varies, for experimental applications in quantum fluids where thermal excitations are always present (statistical mechanics problem)

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THANKS FOR YOUR ATTENTION!

Joint works with: Alberto Villois and Giorgio Krstulovic

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