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FLYING IN A SUPERFLUID

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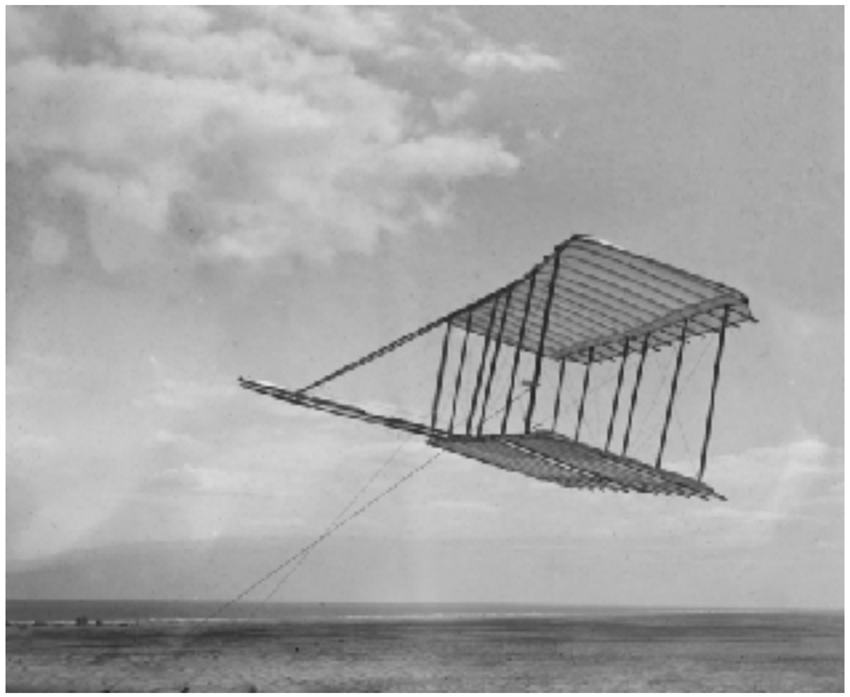
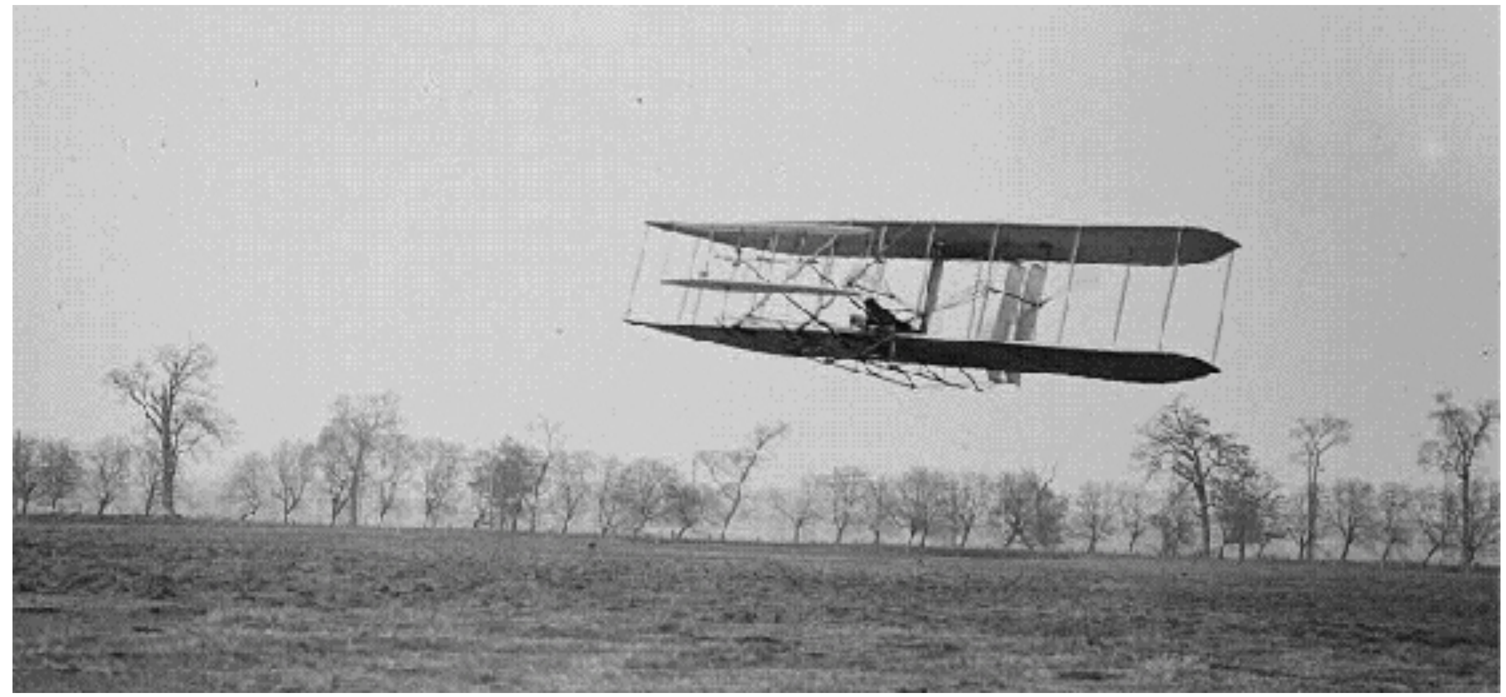
FLYING IN A SUPERFLUID

- Recap on classical theory of flight: 2D and 3D
- Moving obstacles in superfluids
- How an airfoil obstacle/potential may affect the superfluid flow

CLASSICAL THEORY OF FLIGHT



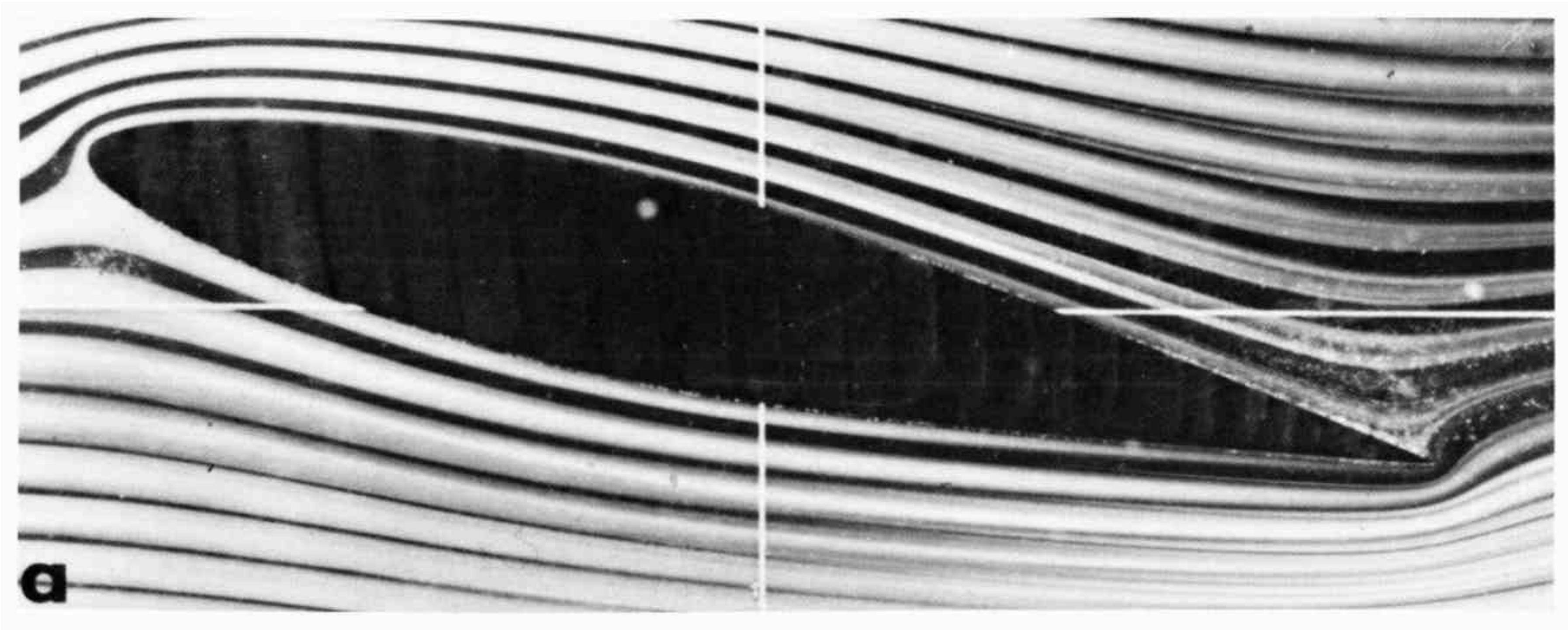


By Wright brothers - Library of Congress, Public Domain [Wikipedia]

- Inviscid theory to predict lift in stationary flow
- Viscous effects to explain the generation of lift and drag effects

[D.J. Achenson, Elementary Fluid Dynamics, Oxford University Press, 1990]

CLASSICAL THEORY OF FLIGHT



[M. Van Dyke, An Album of fluid Motion, 1982]

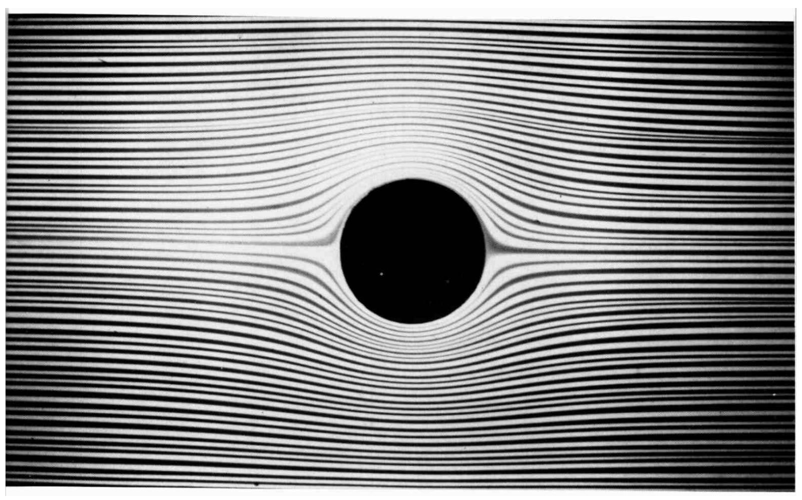
- The due to the positive angle of attack (or geometry) the fluid's speed is higher in the upper part of the airfoil (wing cross-section)
- The lift is a direct consequence of Bernoulli equation

$$\frac{1}{2}|\mathbf{v}|^2 + \frac{p}{\rho} = \text{const.}$$

2D INVISCID THEORY FOR AN AIRFOIL

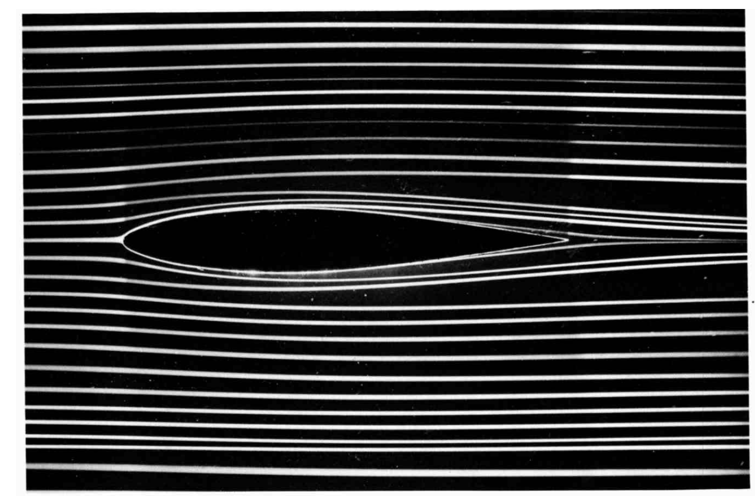
The two-dimensional flow resulting from the incompressible Euler equation past an airfoil can be analytically solved using conformal mapping

[M.Van Dyke, An Album of fluid Motion, 1982]



$$\rightarrow Z(z) \rightarrow$$

$$Z(z) = z + \frac{a^2}{z}$$

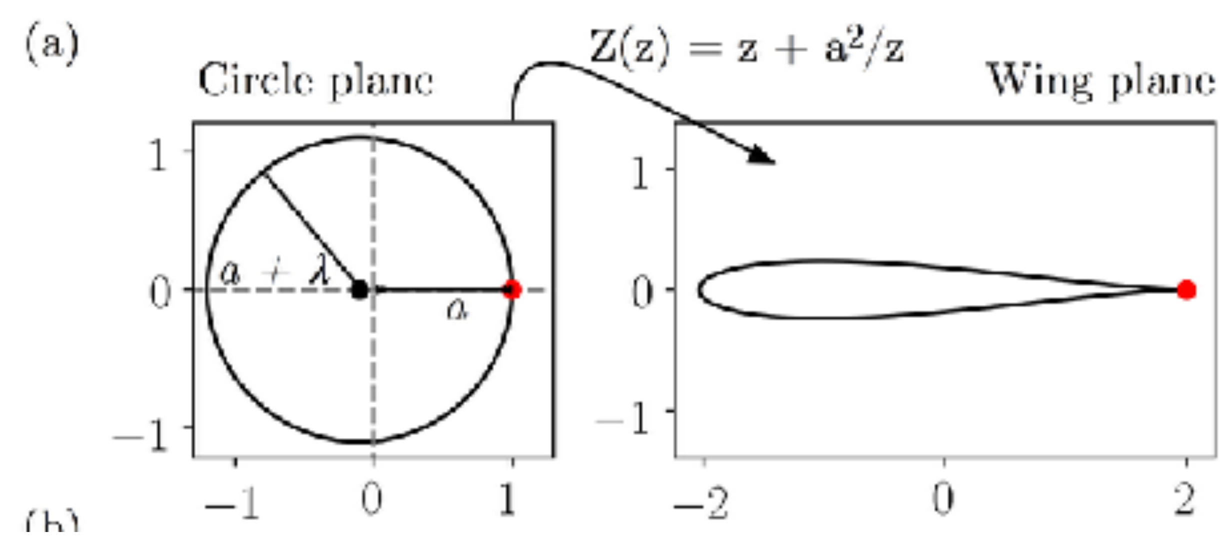


$$\frac{dw}{dz} = U_\infty \left(1 - \frac{a^2}{z^2}\right) - \frac{i\Gamma}{2\pi z}$$

- Complex velocity potential, solution of the flow past a cylinder

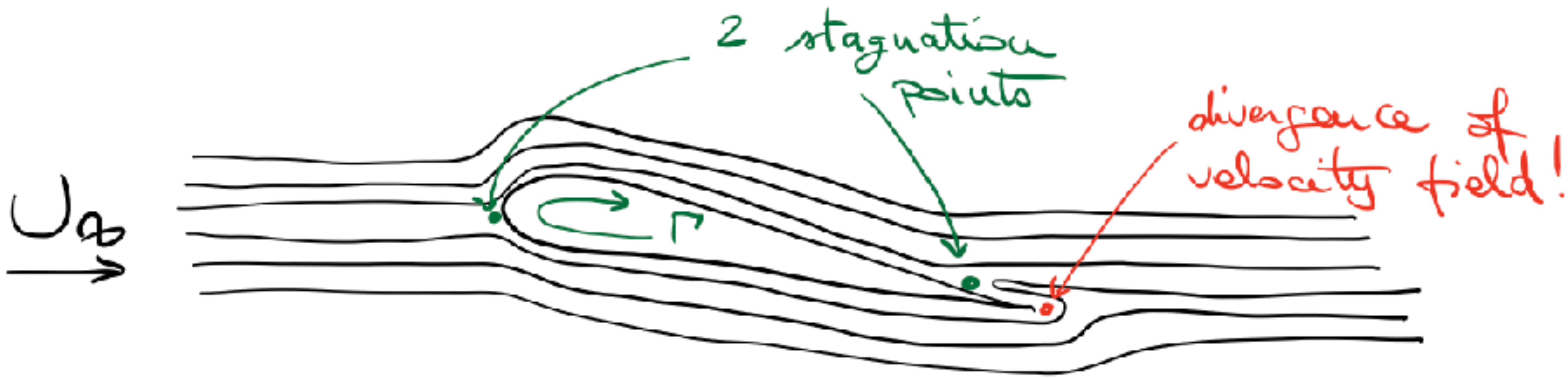
- Joukowski map, example mapping a circle onto an airfoil

  here $\lambda = -0.1, a = 1$



2D INVISCID THEORY FOR AN AIRFOIL

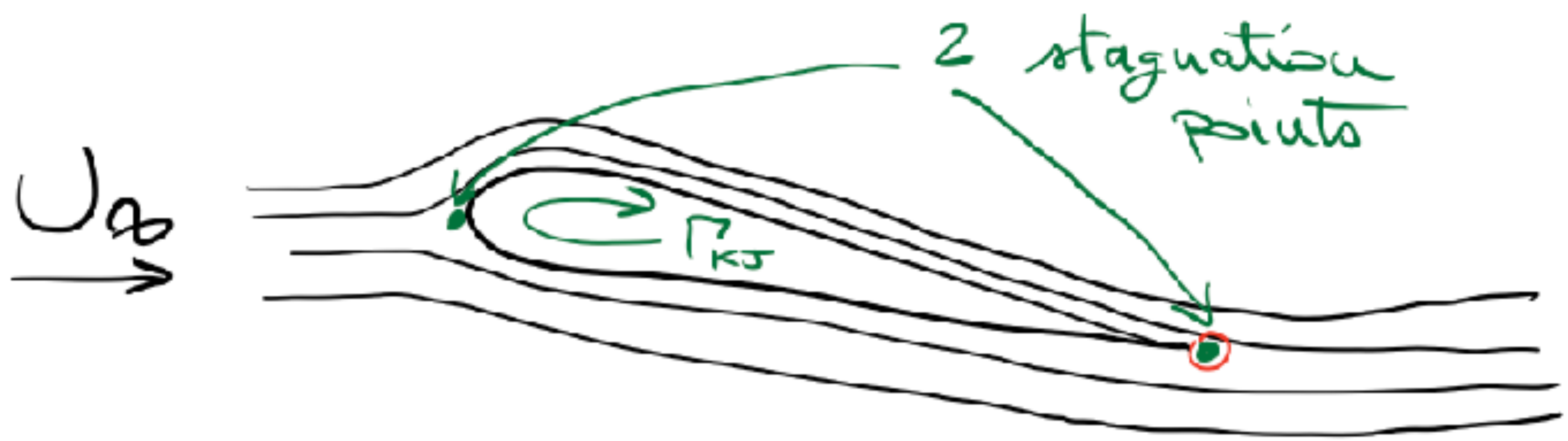
For a generic value of the terminal velocity, angle of attack, airfoil size and circulation around the airfoil, the streamlines in stationary conditions can be sketched as follows



- Two stagnation points (zero speed) at the airfoil, whose positions depend on the value of the circulation around the airfoil
- A divergence of the fluid's speed at the trailing edge of the airfoil due to the presence of a cusp

THE KUTTA-JOUKOWSKI CONDITION

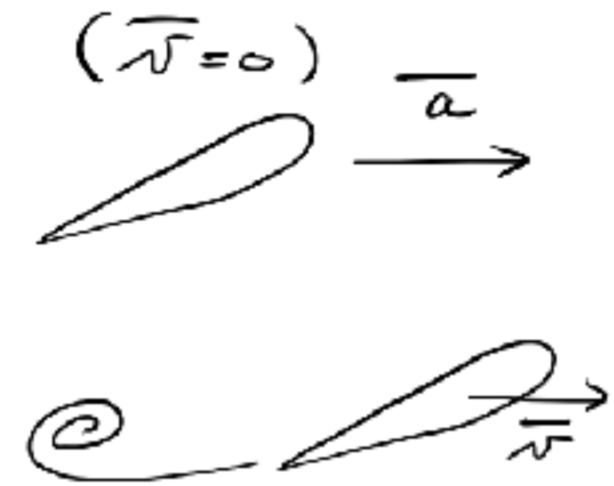
For a generic value of the terminal velocity, angle of attack, airfoil size and circulation around the airfoil, the streamlines in stationary conditions can be sketched as follows



The unphysical divergence of the fluid speed is cancelled by letting one of the two stagnation points meeting the trailing edge. This mathematically results in the Kutta-Joukowski (KJ) condition

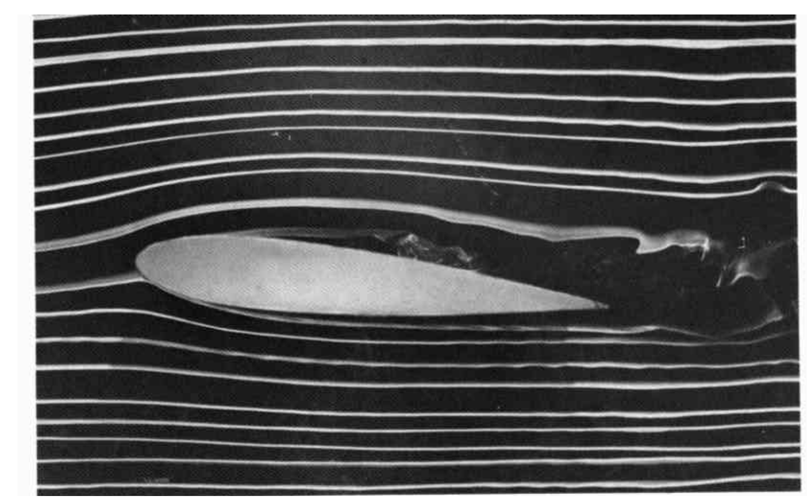
$$\Gamma_{KJ} = 4\pi U_\infty (a + \lambda)\ \sin\alpha$$

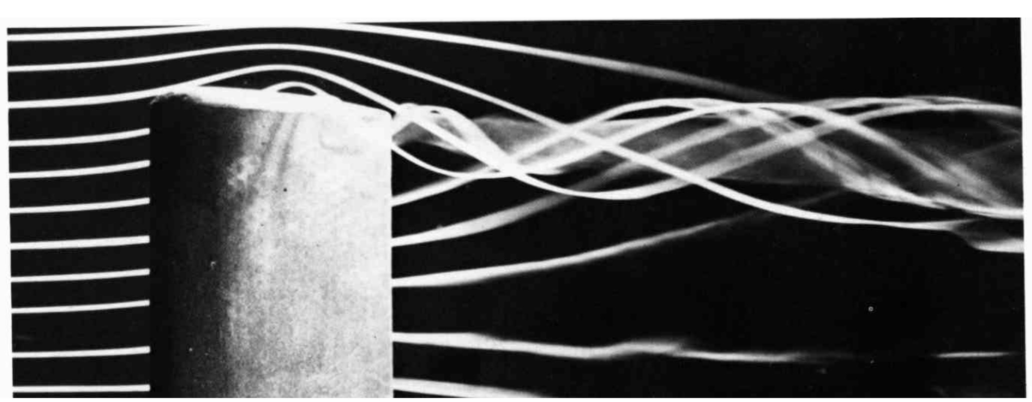
ADDING VISCOUS EFFECTS AND 3D CASE

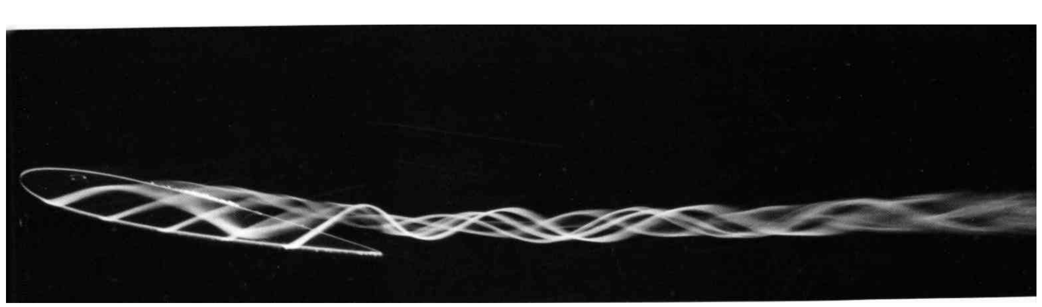


Viscous effects:

- cause generation of the KJ circulation around the airfoil (forbidden in inviscid fluid due to Helmoltz's third theorem)
- responsible for drag forces (form drag and skin drag)
- responsible for stall effect due to detachment of boundary layer







3D case:

- Vortex tubes created at the tips of the wings

Here not considered, only 2D!

FLYING IN A SUPERFLUID

(A superfluid is a fluid where viscosity is identically zero, like a superconductor is a material where electrical resistance is zero.)

- Can an accelerated airfoil acquire circulation?
- If so, what are the admissible values of the lift for a given airfoil, angle of attack and terminal velocity?
- Does the airfoil experience any drag?

THE GROSS-PITAEVSKII MODEL

$$\imath\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g\,|\psi|^2\psi-V_{ext}\psi=0$$

- It is a mean-field equation that can be formally derived to model dilute Bose gases in the limit of zero temperature
- It also model qualitatively well other superfluids like liquid Helium below the λ-point
- This model is nothing but a nonlinear Schroedinger equation, where $\psi(\mathbf{r},t)$ is a complex function describing the order parameter of the system
- m is the mass of each boson, $\hbar$ is the reduced Planck's constant, g weight the effective binary collisions between the bosons, V_{ext} is some external potential

THE GROSS-PITAEVSKII MODEL

$$\imath\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g\,|\psi|^2\psi-V_{ext}\psi=0$$

Using Madelung transformation $\psi=\sqrt{\rho}\exp(\imath\phi)$ and defining density and velocity as $\rho=m\,|\psi|^2$ and $\mathbf{v}=\hbar/m\nabla\phi$, respectively, then

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{u})=0$$

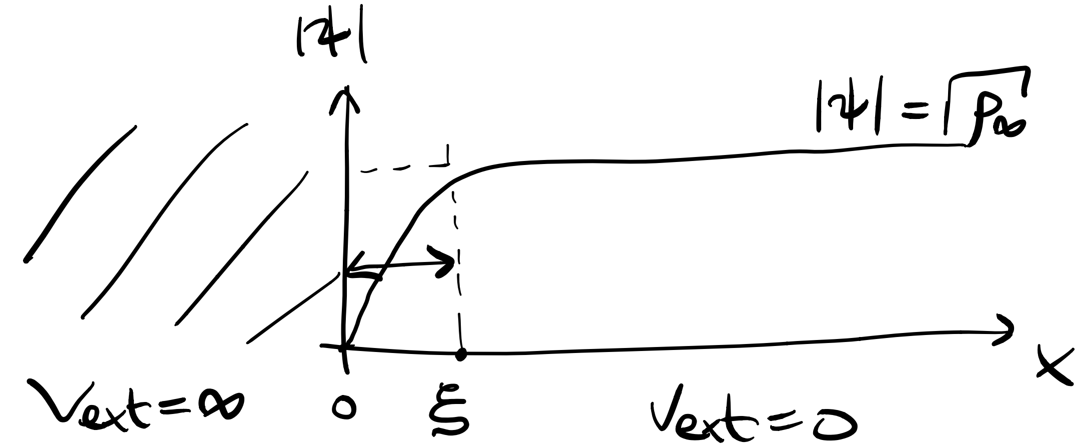
$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=\nabla\left[-\frac{g}{m}\rho+\frac{1}{m}V_{ext}+\frac{\hbar^2}{2m^2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right]$$

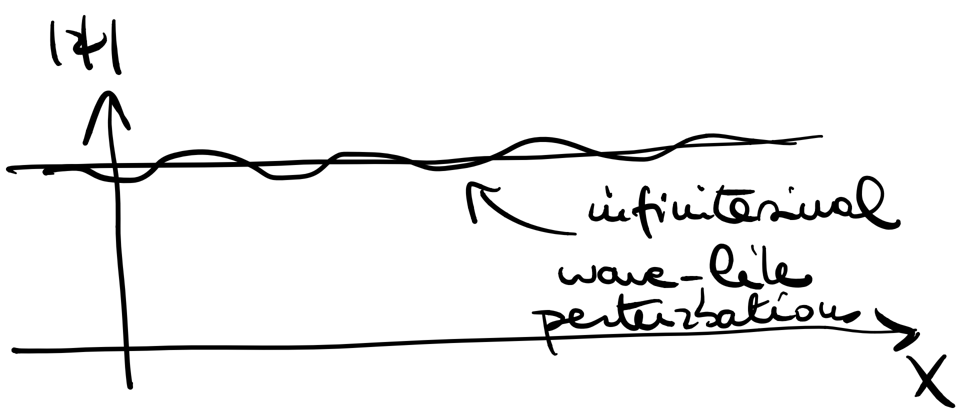
- The GP models an inviscid, barotropic, and irrotational fluid
- The last term of the second equation, the quantum pressure, becomes negligible at scales larger than the healing length ξ

THE GROSS-PITAEVSKII MODEL

$$\imath\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g\,|\psi|^2\psi-V_{ext}\psi=0$$

- In absence of the external potential, the lowest energy state (ground-state) is the uniform state $|\psi_{GS}|=\sqrt{\rho_\infty}$, where ρ_∞ is the bulk density of the fluid
- The healing length $\xi=\sqrt{\hbar^2/(2mg\rho_\infty)}$ is the only inherent length scale of the system



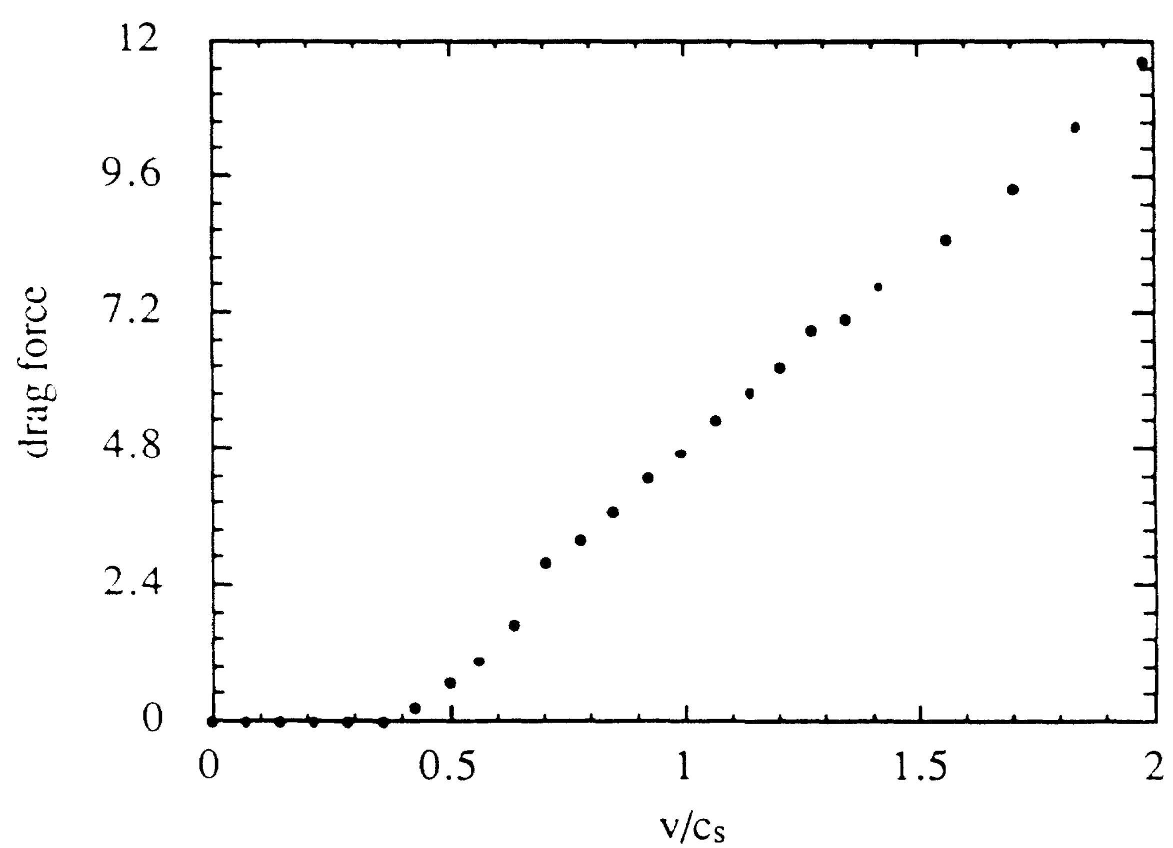


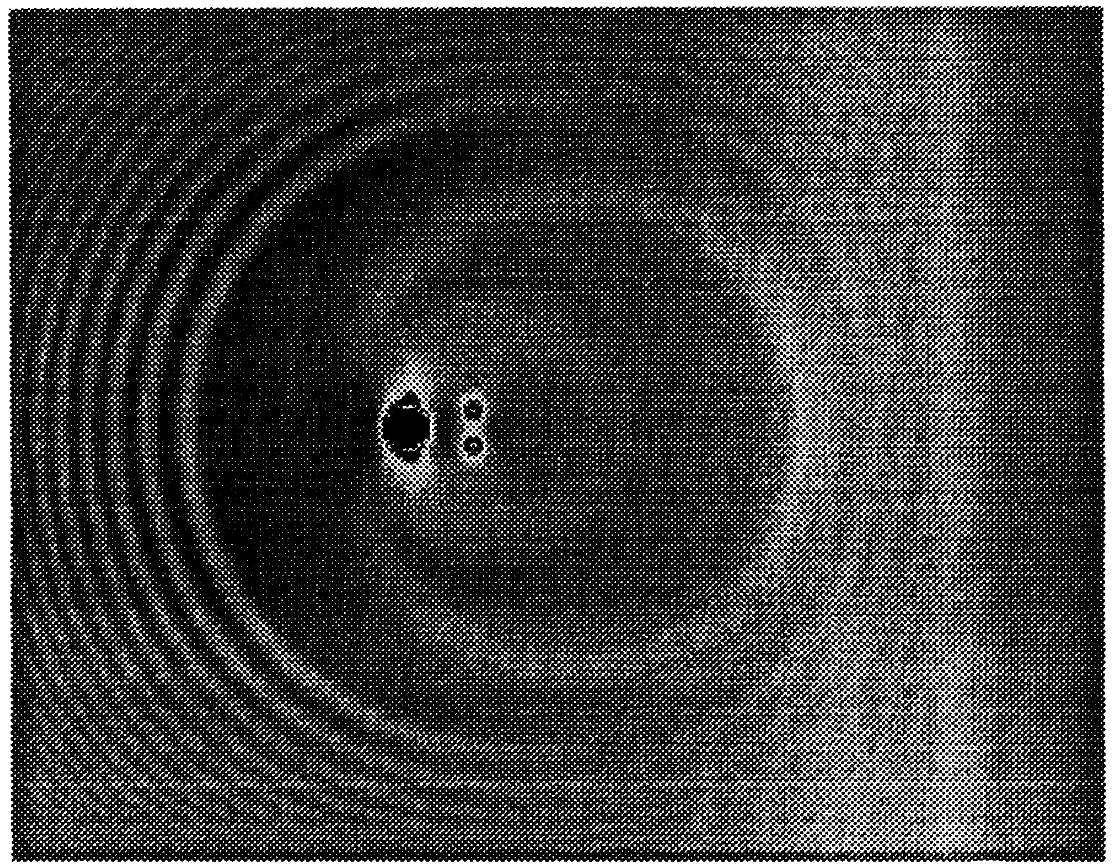
- The infinitesimal perturbations of the ground-state are at large scale phonon-like excitations having speed of sound $c=\sqrt{g\rho_\infty/m}$

EXTERNAL POTENTIAL CYLINDER MOVING IN GP

An external potential moving in a superfluid may cause the flow to break the Landau's critical velocity (sound speed in GP), to generate excitations eventually causing dissipation

<u>2d cylinder</u>





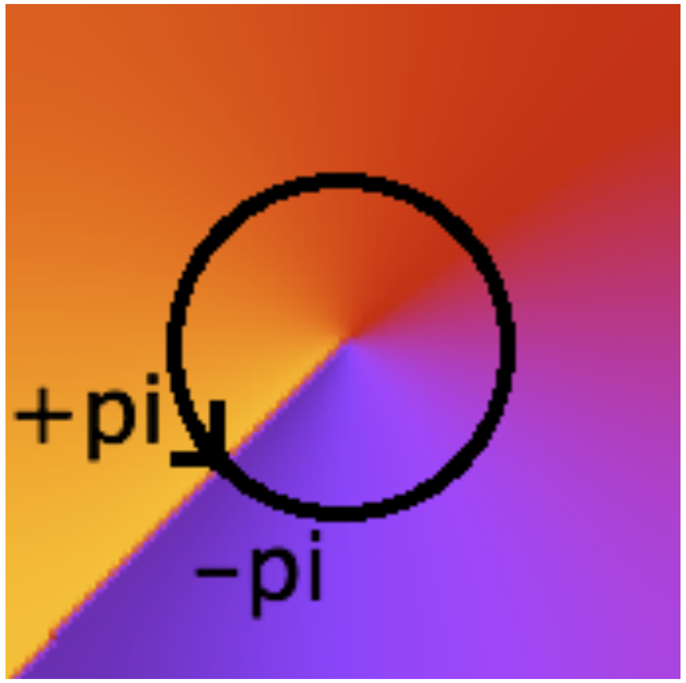
[Frisch et al., PRL 69, 1644 (1992)]

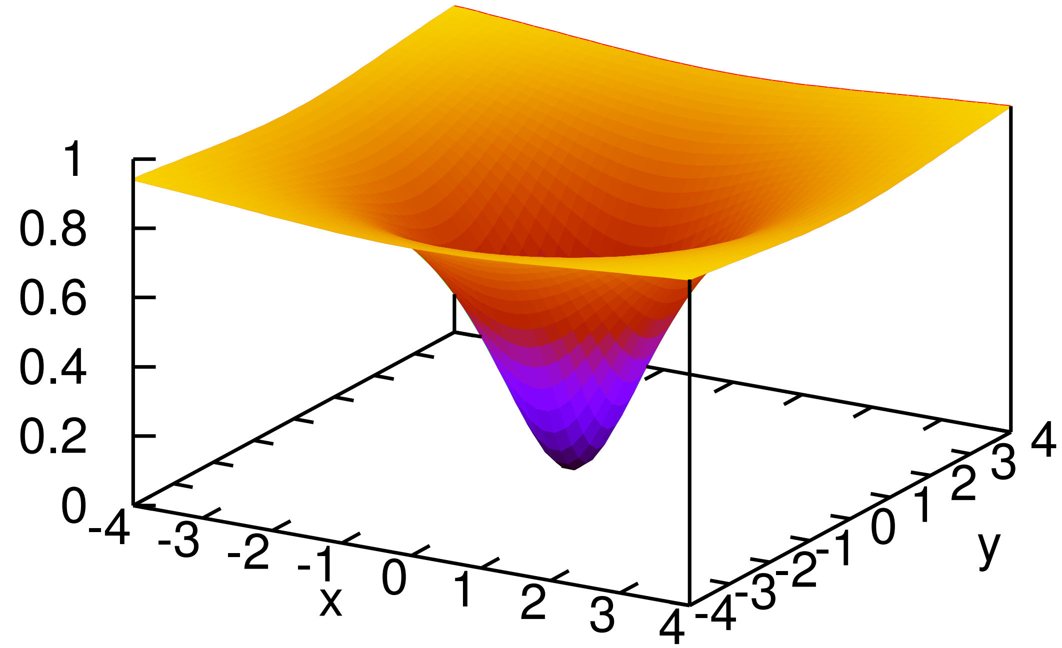
THE GROSS-PITAEVSKII MODEL

$$\imath\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi-g\,|\psi|^2\psi-V_{ext}\psi=0$$

Using Madelung transformation $\psi=\sqrt{\rho}\,\exp(\imath\phi)$ and defining density and velocity as $\rho=m\,|\psi|^2$ and $\mathbf{v}=\hbar/m\nabla\phi$, respectively, then

- Vortices are topological defect of the wave-function's argument
- Each vortex has circulation $\Gamma=\oint\mathbf{v}\cdot d\mathbf{l}=n\frac{\hbar}{m}\Delta\phi=n\kappa$





EXTERNAL POTENTIAL MOVING IN GP

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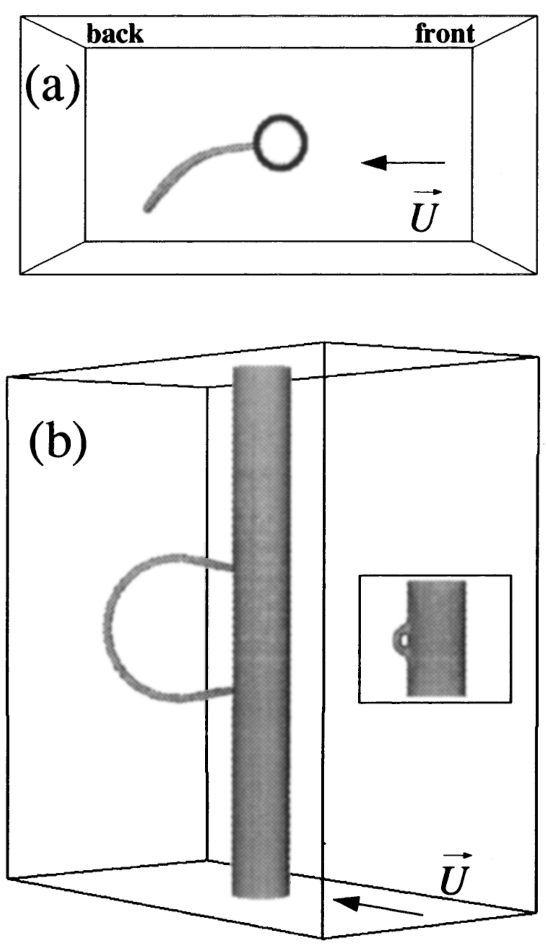
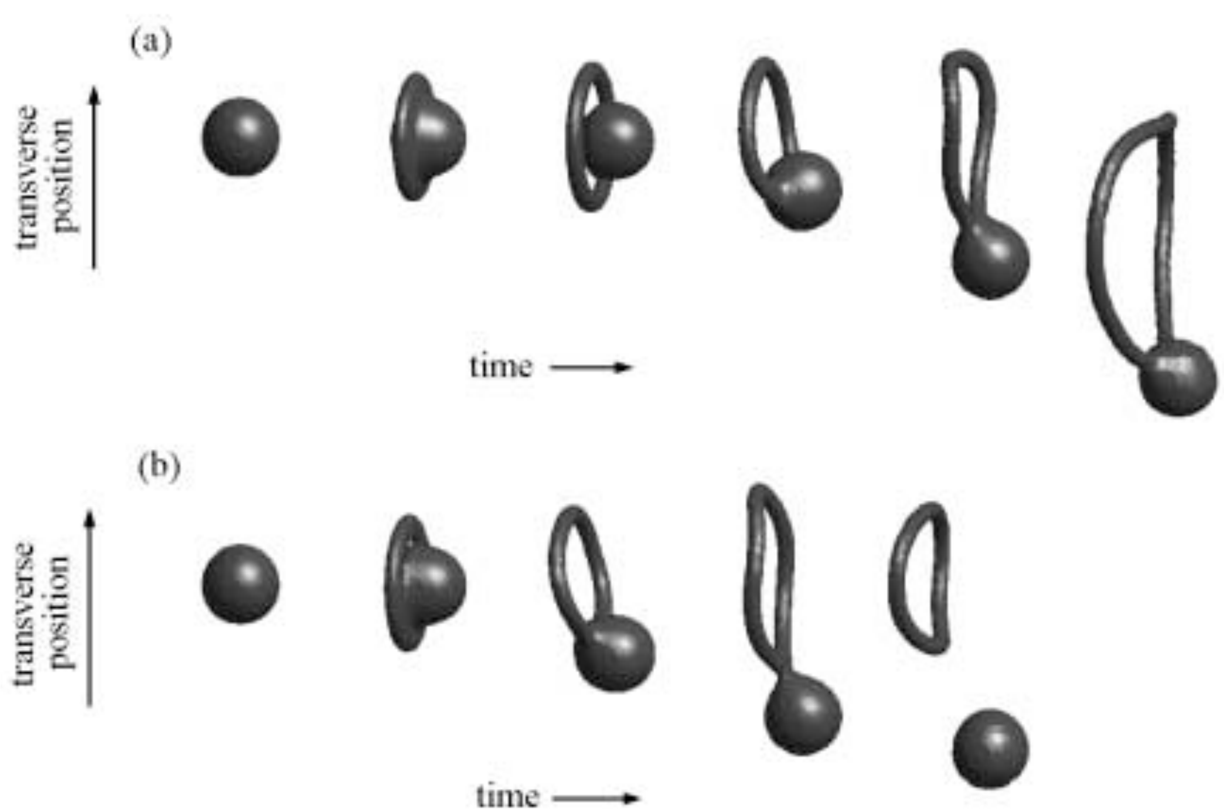
3d cylinder



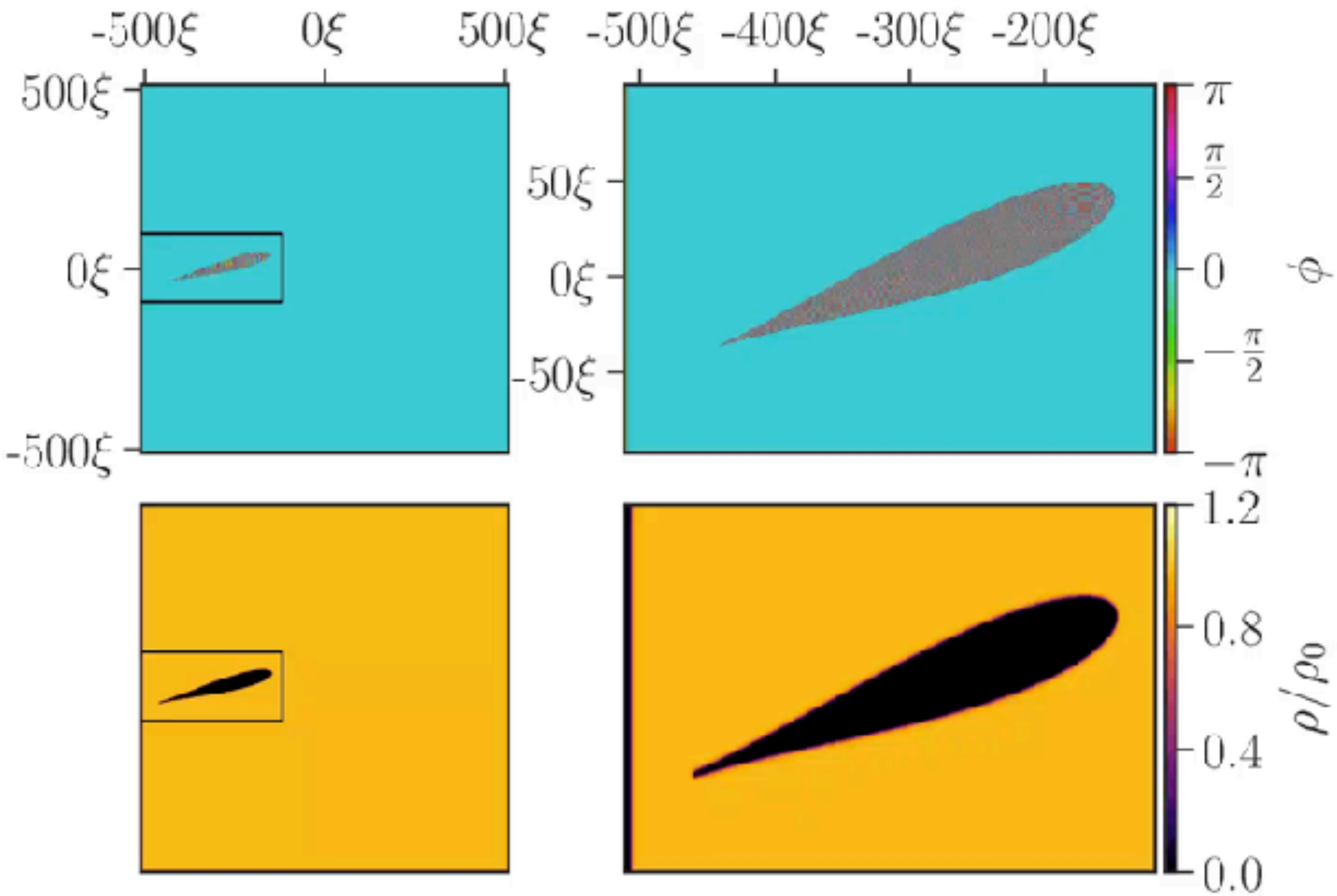
[Nore et al., PRL 84, 2191 (2000)]

3d sphere



[Winiecki & Adams, Europhys. Lett. 52, 257-263 (2000)]

A TYPICAL SIMULATION



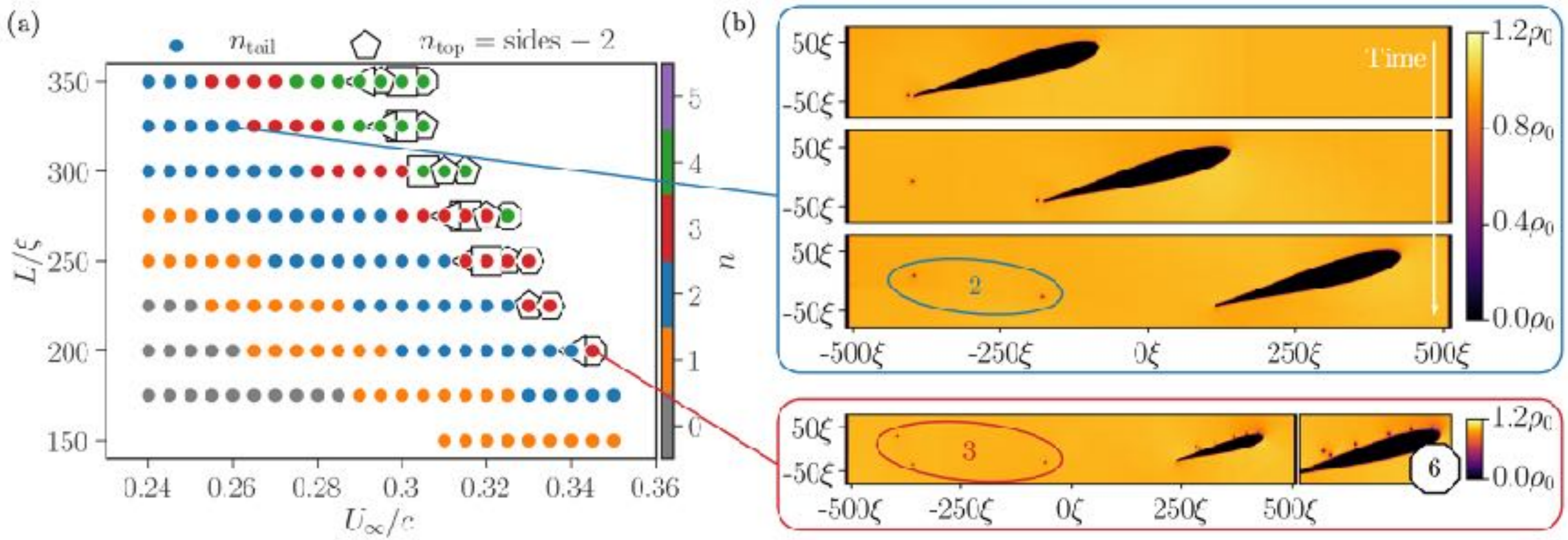
Top: evolution of the phase field.

Bottom: evolution of the superfluid density field.

- The airfoil moves initially with constant acceleration until it reaches a terminal velocity $U_\infty = 0.29c$
- The airfoil's length is $L = 325\xi$ and angle of attack $\alpha = \pi/12$
- Confining potential at the end of the computational box

EXPLORATION OF THE PARAMETERS SPACE

- We vary both the airfoil length and terminal velocity
- The airfoil shape ($\lambda = 0.1$) and angle of attack $\alpha = \pi/12$ are constant



Left: number of vortices produced at the trailing edge. Vortices produced at the top are highlighted with a polygon. Right: two simulation examples, the latter with the detachment of the boundary layer causing a stall condition.

HOW TO PREDICT THE NUMBER OF VORTICES GENERATED?

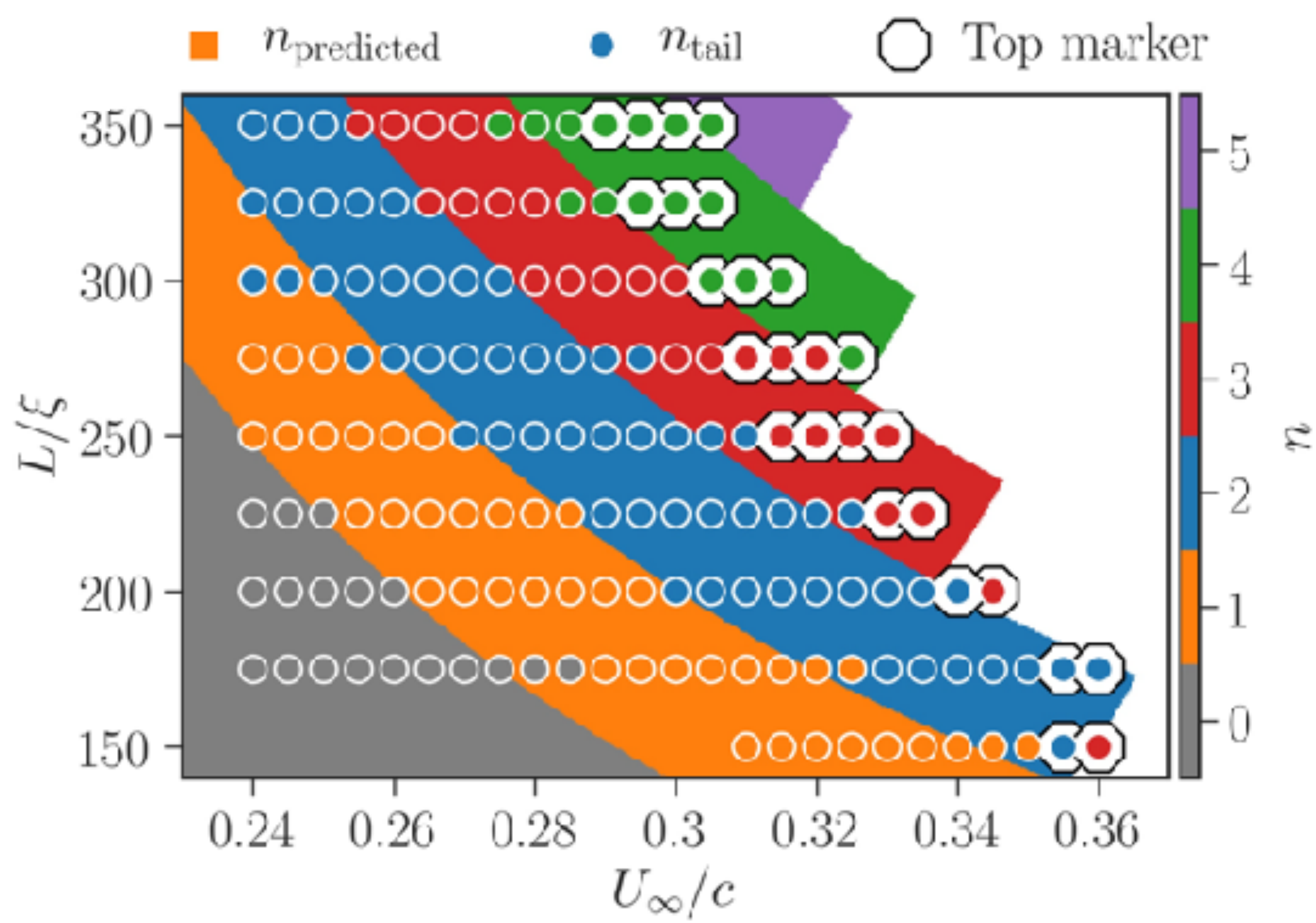
Introducing a dispersive boundary layer with thickness $r = C\xi$

$$C \leq \frac{3}{8}\frac{L}{\xi}\left(\frac{U_\infty}{c}\right)^2 \sin^2(\alpha)\left(1 - \frac{\Gamma}{\Gamma_{KJ}}\right)^2$$

where $\Gamma = n\kappa$, with $n \in \mathbb{N}$
and Γ_{KJ} is the KJ condition

best fit gives $C \approx 0.55$

Number of vortices generated depending on the speed and length parameters. The curves indicate the phenomenological prediction. The white area indicate the stalling behaviour.

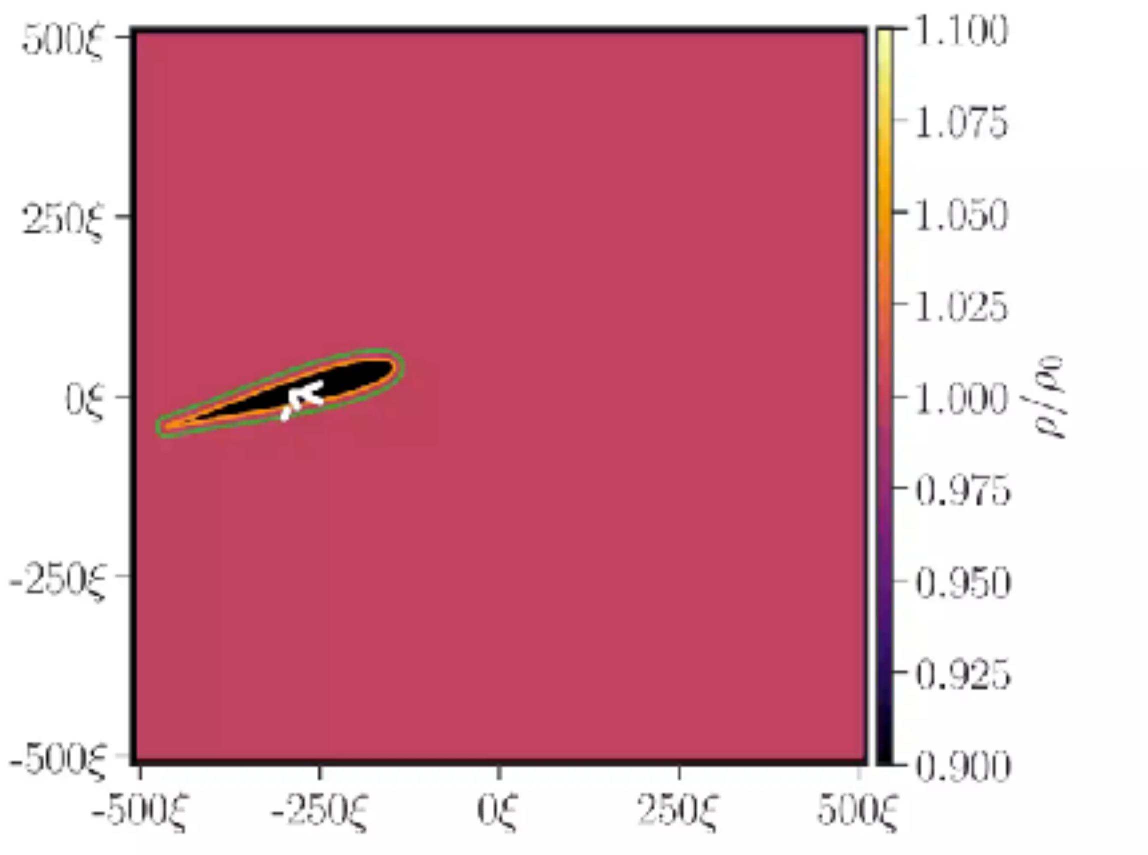


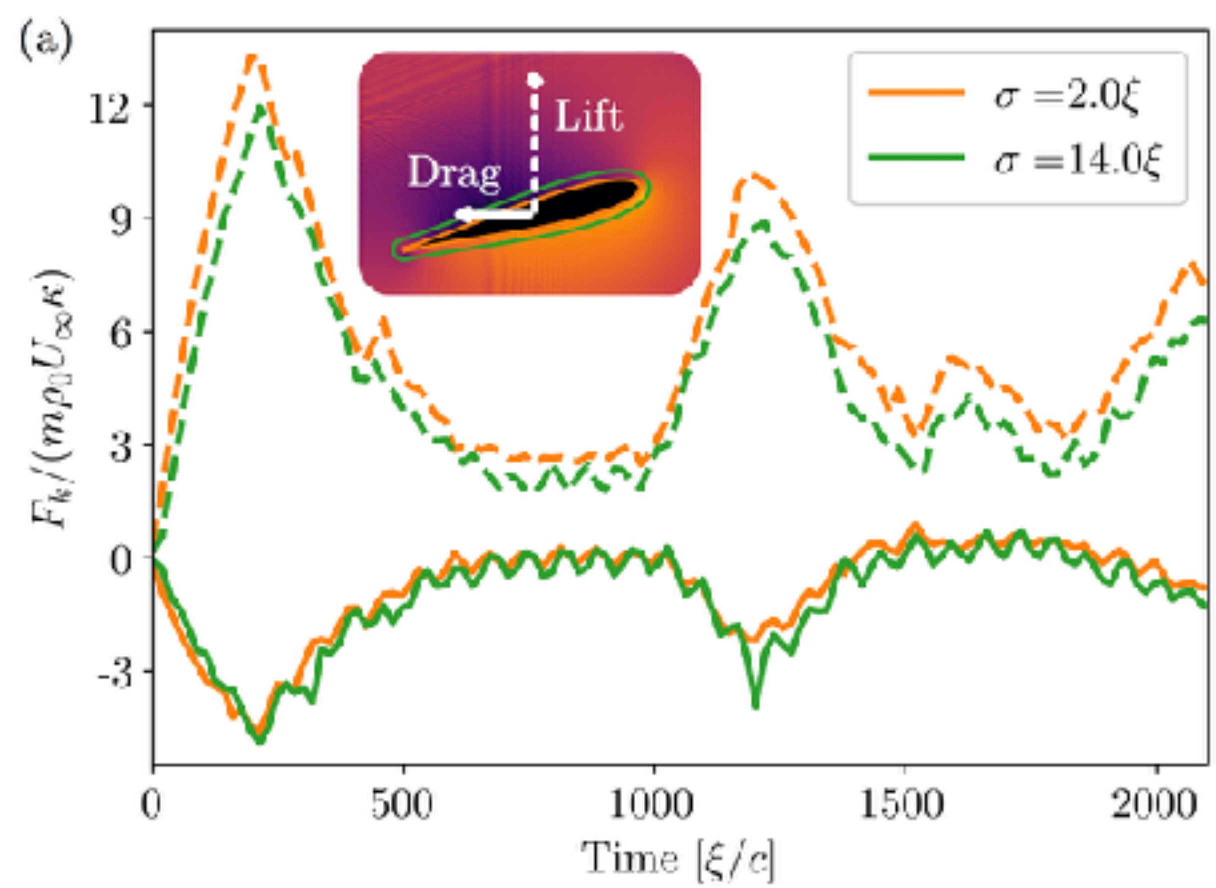
ABOUT LIFT AND DRAG

Lift and drag is obtained integrating the stress-energy tensor

$$F_k = -\oint_{\mathscr{C}} T_{jk}\, n_j\, d\ell\,, \quad \text{where} \quad T_{jk} = m\rho u_j u_k + \frac{1}{2}\delta_{jk} g \rho^2 - \frac{\hbar^2}{4m}\rho \partial_j \partial_k \ln \rho$$

$\mathscr{C}$ closed path containing the airfoil

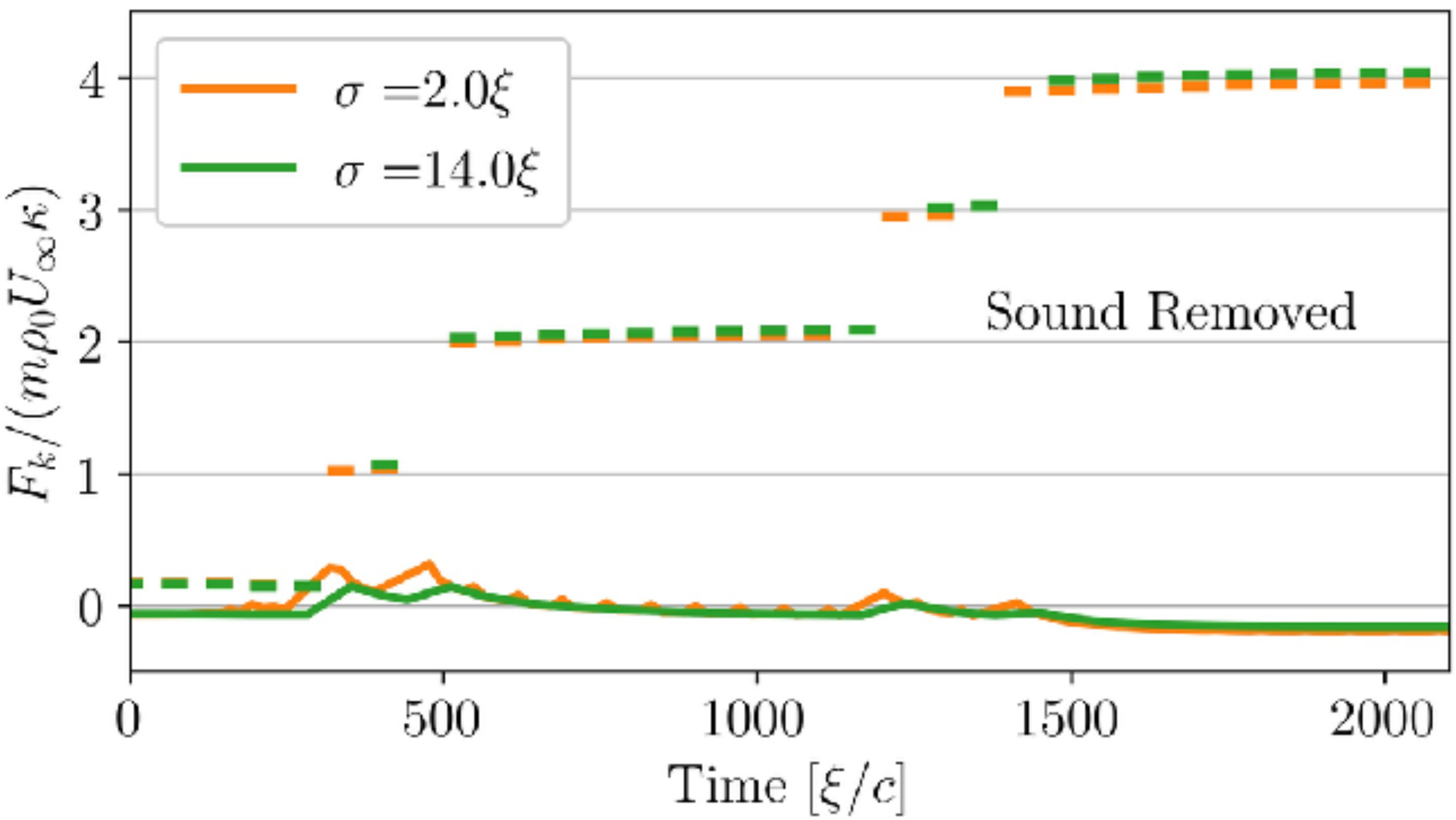




Left: video showing the sound emission during the vortex nucleation at the trailing edge. Right: rescaled lift (dashed) and drag (solid) versus time computed for different contours around the airfoil.

ABOUT LIFT AND DRAG (SOUND FILTERED)

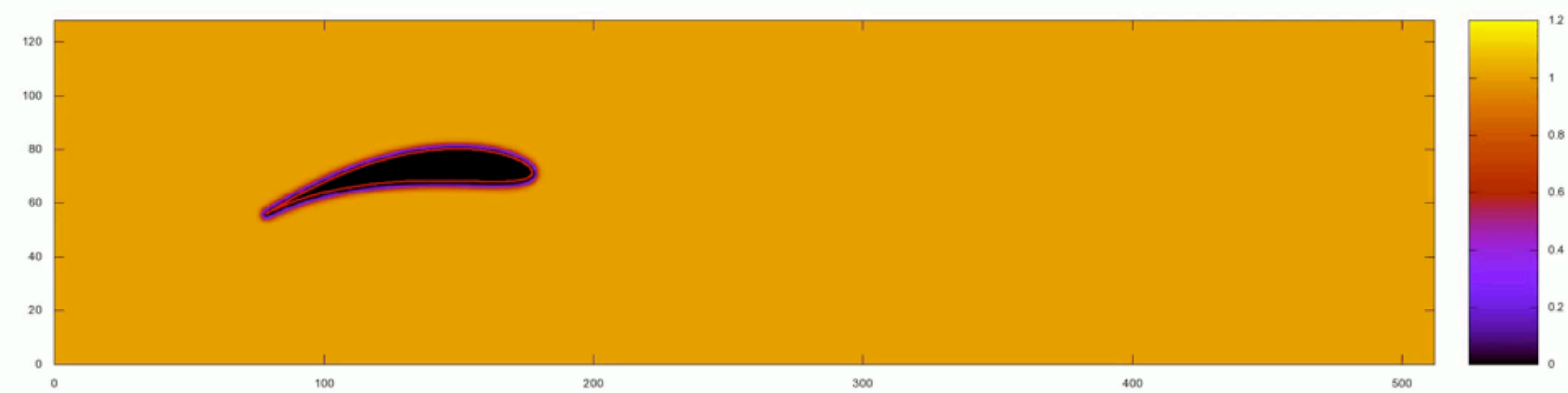
- filter the acoustic component in the velocity field
- use density field prescribed by the stationary Bernoulli equation



Rescaled lift (dashed) and drag (solid) versus time computed for different contours around the airfoil removing sound

Lift appears now quantised and drag becomes nearly zero after the vortex nucleation

CONCLUSIONS



- An airfoil moving in a superfluid can generate vortices at the trailing edge by breaking the Landau's critical speed
- To preserve the total circulation, the airfoil acquires a non-zero circulation
- This process is unsteady and generates sound
- When sound is removed (or steady regime is achieved) the airfoil experiences a quantised lift and no drag)

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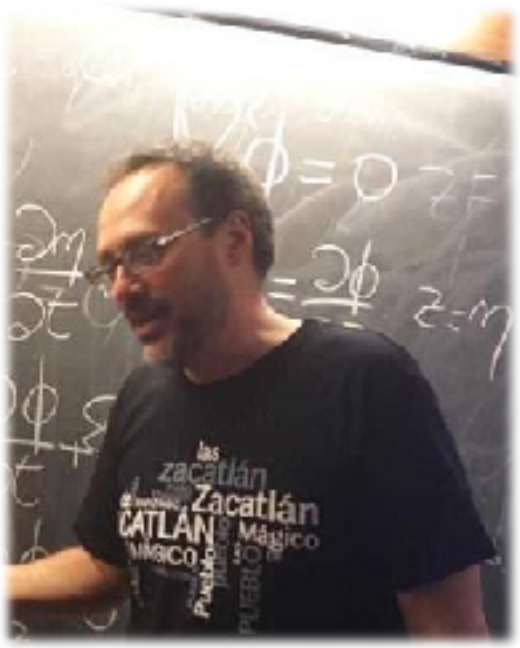
THANKS FOR YOUR ATTENTION!

Joint work with: Seth Musser, D.P., Miguel Onorato, William T.M. Irvine



Acknowledgments

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